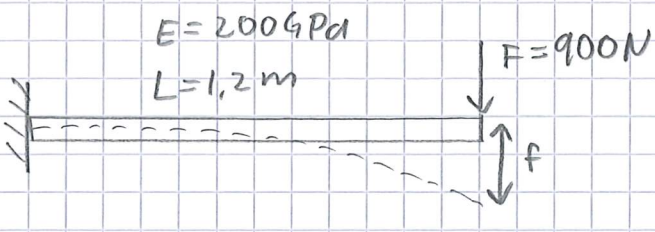
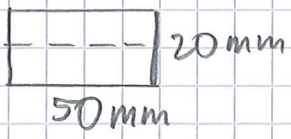
①

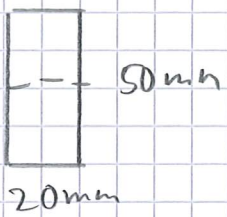


a)



$$f = \frac{F \cdot L^3}{3EI} = \frac{F \cdot L^3}{3E \cdot \frac{b \cdot h^3}{12}} = \frac{900 \cdot 1200^3}{3 \cdot 200000 \cdot \frac{50 \cdot 20^3}{12}} \approx 78\,mm$$

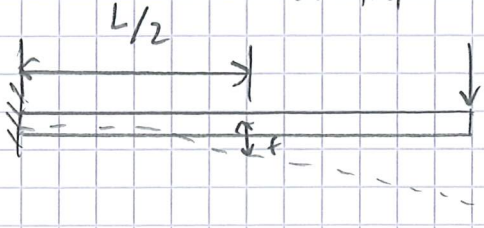
b)

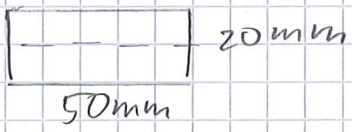


$$f = \frac{F \cdot L^3}{3E \frac{b \cdot h^3}{12}} = \frac{900 \cdot 1200^3}{3 \cdot 200000 \cdot \frac{20 \cdot 50^3}{12}} \approx 12,5\,mm$$

(2)

$L = 1{,}2\ m$
$E = 200\ GPa$





a

$$f(x) = \frac{FL^3}{6EI}\left(3\frac{x^2}{L^2} - \frac{x^3}{L^3}\right)$$

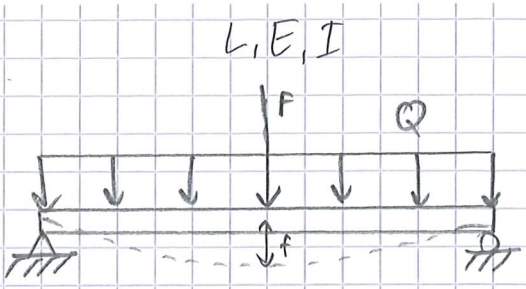
$$f\left(\frac{L}{2}\right) = \frac{F \cdot L^3}{6EI}\left(3\frac{\left(\frac{L}{2}\right)^2}{L^2} - \frac{\left(\frac{L}{2}\right)^3}{L^3}\right) = \frac{FL^3}{6EI}\left(\frac{3}{4} - \frac{1}{8}\right) = \frac{5FL^3}{48EI}$$

$$f = \frac{5FL^3}{48EI} = \frac{5 \cdot 900 \cdot 1200^3}{48 \cdot 200000 \cdot \frac{50 \cdot 20^3}{12}} = 24{,}3\ mm$$

Generellt

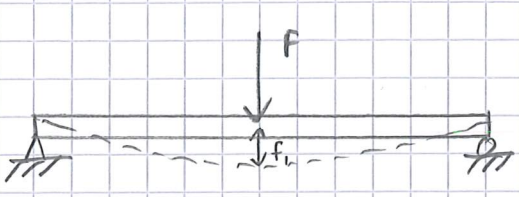
$$f\left(\frac{L}{2}\right) = \frac{5 \cdot FL^3}{48\,EI}$$

③



• Uppdelning i elementarfall ger:

①



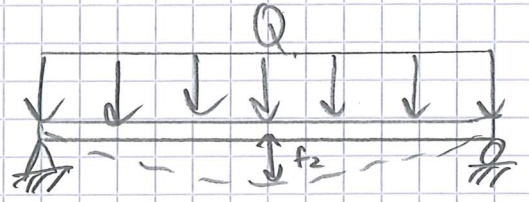
$$f(x) = \frac{F \cdot b \cdot x (L^2 - b^2 - x^2)}{6EIL}$$

Vid $x = \frac{L}{2}$ ges $f\left(\frac{L}{2}\right) = \frac{F \cdot \left(\frac{L}{2}\right) \cdot \left(\frac{L}{2}\right)\left(L^2 - \left(\frac{L}{2}\right)^2 - \left(\frac{L}{2}\right)^2\right)}{6EI \cdot L} =$

$$= \frac{F \cdot \frac{L^2}{4}\left(\frac{4L^2}{4} - \frac{L^2}{4} - \frac{L^2}{4}\right)}{6EI \cdot L} = \frac{F \cdot \frac{2L^4}{16}}{6EI \cdot L} = \frac{FL^3}{48EI}$$

$$\Rightarrow f_1 = \frac{FL^3}{48EI}$$

②

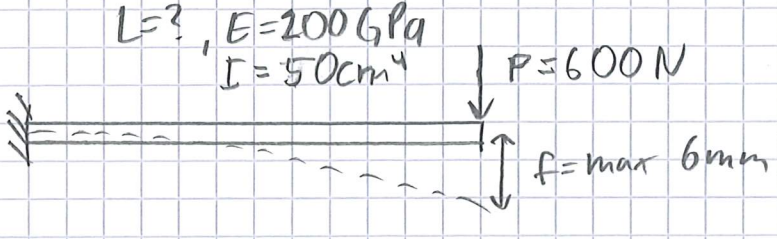


$$f_2 = \frac{5QL^3}{384EI}$$

Totala utböjningen blir:

$$f = f_1 + f_2 = \frac{FL^3}{48EI} + \frac{5QL^3}{384EI} = \frac{L^3}{EI}\left(\frac{F}{48} + \frac{5Q}{384}\right)$$

(4)

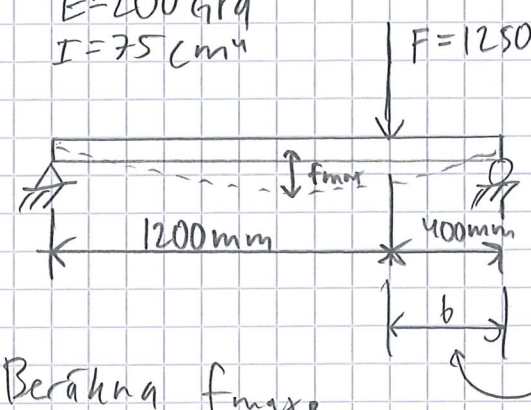


$$f = \frac{FL^3}{3EI}$$

$$L = \left(\frac{3EI \cdot f}{F}\right)^{1/3} = \left(\frac{3 \cdot 200\,000 \cdot 500\,000 \cdot 6}{600}\right)^{1/3} \approx 1442\,mm$$

⑤

$E = 200\ GPa$
$I = 75\ cm^4$



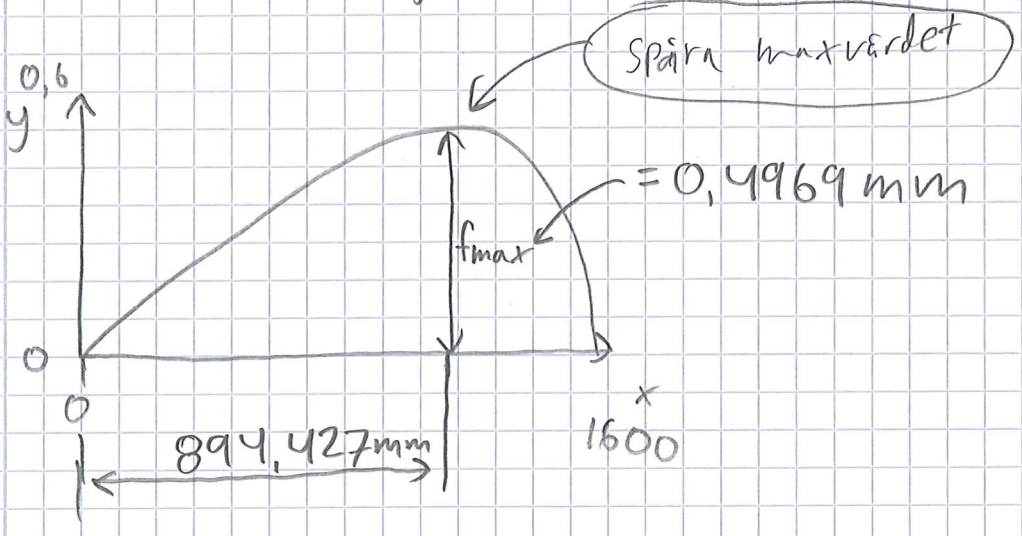
enligt formelsamling

Beräkna f_{max}.

$$f(x) = \frac{F b x (L^2 - b^2 - x^2)}{6EIL}$$ där $x = ?$, $b = 400\ mm$ (för $x < 1200\ mm$)

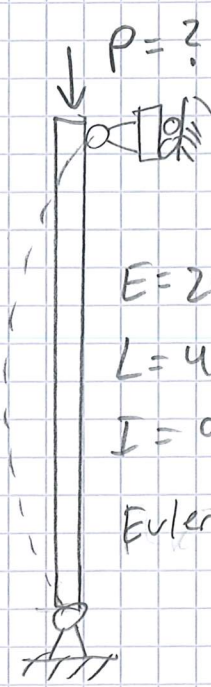
$$f(x) = \frac{1250 \cdot 400 \cdot x (1600^2 - 400^2 - x^2)}{6 \cdot 200\,000 \cdot 750\,000 \cdot 1600}$$

maxvärdet ges genom grafritning:



Svar: f_{max} är cirka 0,50 mm vid $x = 895\ mm$.

(6)



- Säkerhet mot knäckning: 7.
- Säkerhet mot sträckning: 3.

- Kontroll av slankhetstal

$$\lambda = \frac{L}{\sqrt{\frac{I}{A}}} = \frac{4000}{\sqrt{\frac{900000}{3000}}} \approx 231 \text{ , ok} > 100!$$

- Eulerfall II (kontroll mot knäckning)

$$F_k = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \cdot 200\,000 \cdot 900000}{4000^2} \approx 111033 \text{ N}$$

$$F_{till} = \frac{F_k}{n_k} = \frac{111033}{7} \approx \underline{\underline{15862 \text{ N}}}$$

- Tryckspänning (kontroll mot spänning)

$$\sigma_{till} = \frac{R_e}{n_s} = \frac{355}{3} \approx 118{,}3 \text{ MPa}$$

$$F = \sigma_{till} \cdot A = 118{,}3 \cdot 3000 = \underline{\underline{355\,000 \text{ N}}}$$ (ej dimensionerande)

(7)

$\phi D / \phi d = \phi 30 / \phi 25$ mm

$E = 70$ GPa, $R_e = 75$ MPa

$L = 1{,}2$ m

Euler-fall II

$$P_K = \frac{\pi^2 E I}{L^2} = \frac{\pi^2 \cdot 70000 \cdot \frac{\pi(30^4 - 25^4)}{64}}{1200^2} \approx 9877 \text{ N}$$

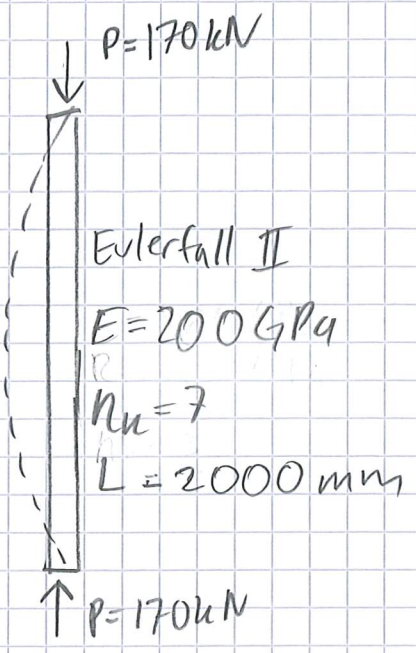
$$P_K = \frac{P}{n_K} \Rightarrow n_K = \frac{P_K}{P} = \frac{9877}{3000} \approx 3{,}3$$

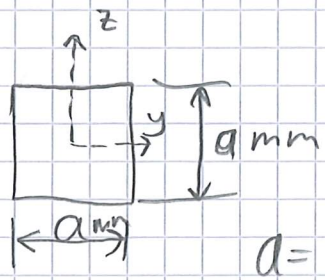
- Tryckspänning

$$\sigma_{max} = \frac{F}{A} = 3000 / \frac{\pi 30^2}{4} - \frac{\pi 25^2}{4} \approx 13{,}9 \text{ MPa}$$

$$n_s = \frac{R_e}{\sigma_{max}} = \frac{75}{13{,}9} \approx 5{,}4$$

(8)





a = ?

$$P \cdot n_k = \frac{\pi^2 E I}{L^2} \Rightarrow I = \frac{P \cdot n_k \cdot L^2}{\pi^2 \cdot E}$$

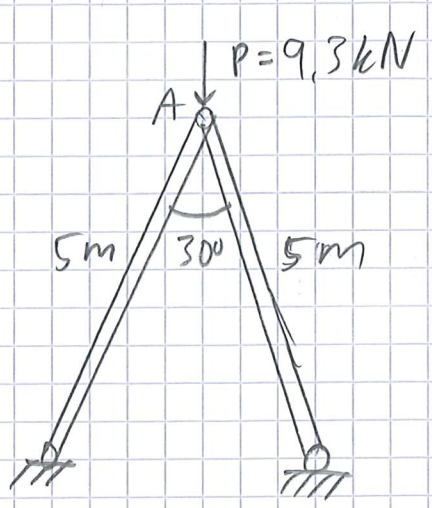
$$I = \frac{b \cdot h^3}{12} \quad \text{nu är } b=h=a \Rightarrow \frac{a \cdot a^3}{12} = \frac{a^4}{12}$$

$$\Rightarrow \frac{a^4}{12} = \frac{P \cdot n_k \cdot L^2}{\pi^2 \cdot E}$$

$$a = \sqrt[4]{\frac{12 \cdot P \cdot n_k \cdot L^2}{\pi^2 \cdot E}} =$$

$$= \left(\frac{12 \cdot 170\,000 \cdot 7 \cdot 2000^2}{\pi^2 \cdot 200\,000} \right)^{1/4} \approx 73{,}34 \text{ mm}$$

⑨

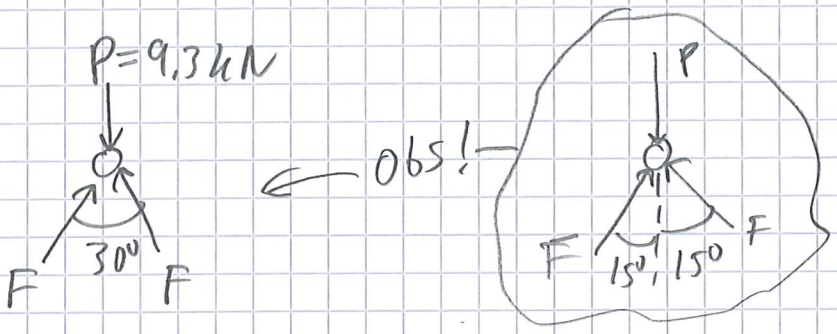


Eulerfall II

$E = 200\ GPa$

$n_k = 5$

Frilägg punkten A



$\uparrow: F \cdot \cos(15°) + F\cos(15°) - P = 0$

$$F = \frac{P}{2\cos(15°)}$$

$$n_k \cdot F = \frac{\pi^2 \cdot EI}{L^2} \quad \text{där } I = \frac{\pi D^4}{64}$$

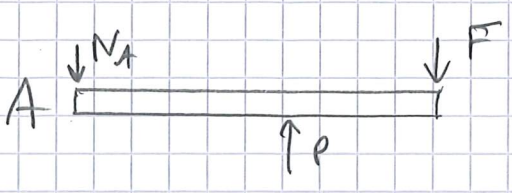
$$\Rightarrow I = \frac{L^2 \cdot n_k \cdot \left(\frac{P}{2 \cdot \cos(15°)}\right)}{\pi^2 \cdot E} = \frac{\pi D^4}{64}$$

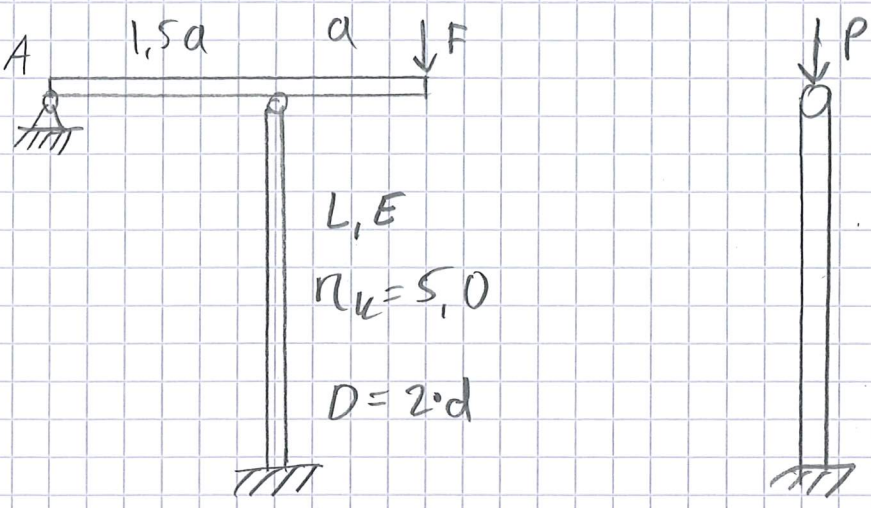
$$\Rightarrow D = \left[\frac{64 \cdot L^2 \cdot n_k \cdot \left(\frac{P}{2 \cdot \cos(15°)}\right)}{\pi^2 \cdot E \cdot \pi}\right]^{1/4}$$

1/2

⑨ $$D = \left[\frac{64 \cdot 5000^2 \cdot 5 \cdot \left(\frac{9300}{2 \cdot \cos(150)} \right)}{\pi^2 \cdot 200000 \cdot \pi} \right]^{1/4} \approx 49{,}92\,mm$$

(10)





$$I = \frac{\pi}{64} \cdot \left[(2d)^4 - d^4 \right] =$$

moment runt A ger kraften P

$$\Rightarrow \overrightarrow{A}: \quad F \cdot (1,5 \cdot a + a) - P \cdot (1,5a) = 0$$

$$F \cdot \left(\frac{5}{2} a \right) - P \left(\frac{3}{2} a \right) = 0$$

$$P = \frac{F \frac{5}{2} a}{\frac{3}{2} a} = \frac{5}{3} F \; N$$

Eulerfall 3

$$F_k = \frac{2,05 \cdot \pi^2 \cdot EI}{L^2}$$

$$I = \frac{F_k \cdot L^2}{2,05 \cdot \pi^2 \cdot E}$$

1 1/2

(10)

$$I = \frac{\pi}{64}\left[16d^4 - d^4\right] = \frac{\pi}{64}\,15d^4 = \frac{15\cdot\pi d^4}{64}$$

$$\frac{15}{64}\pi d^4 = \frac{F_k \cdot L^2}{2{,}05\cdot\pi^2\cdot E}$$

$$d = \left[\frac{64\cdot F_k\cdot L^2}{15\cdot\pi\cdot 2{,}05\cdot\pi^2\cdot E}\right]^{1/4} \quad \text{där } F_k = \frac{5}{3}F\cdot\eta_k$$