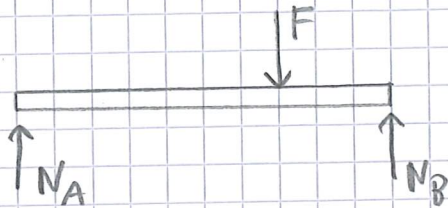
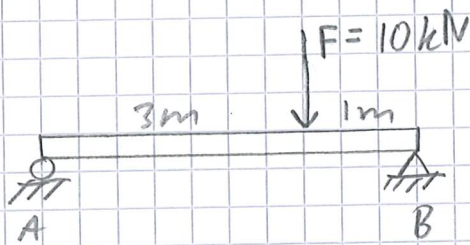


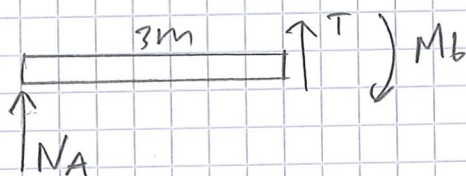
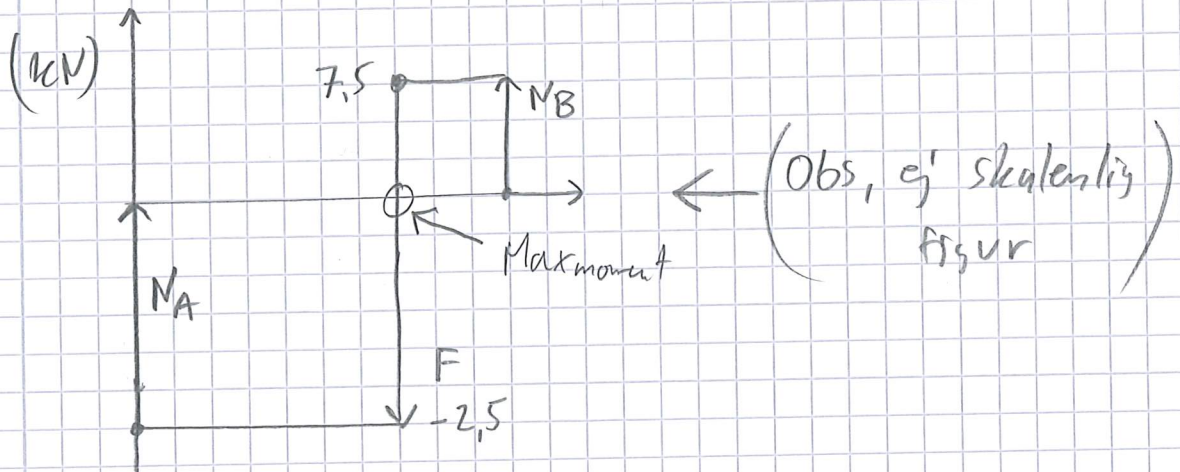
①



$$\begin{cases} \uparrow: N_A + N_B - F = 0 \\ \overset{\curvearrowright}{A}: F \cdot 3 - N_B \cdot 4 = 0 \end{cases}$$

$$\Rightarrow N_A = 2,5 \text{ kN} \text{ och } N_B = 7,5 \text{ kN}$$

Trätkraftsdiagram

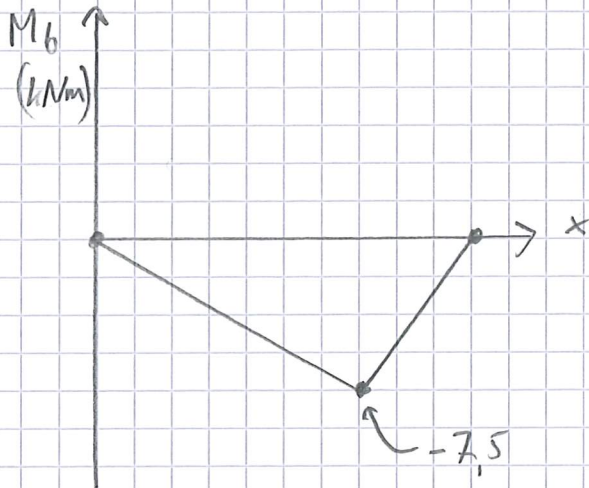
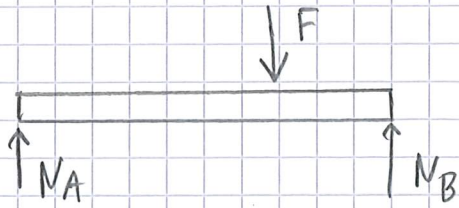


$$\overset{\curvearrowright}{T}: N_A \cdot 3 + M_b = 0 \Rightarrow M_b = -3 \cdot N_A = -3 \cdot 2,5 = -7,5 \text{ kNm}$$

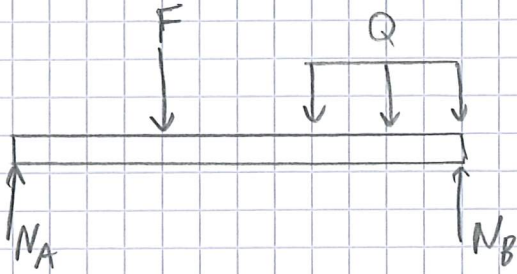
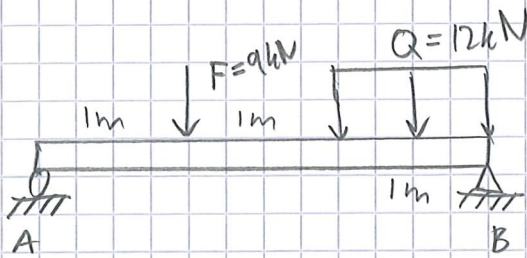
1/2

①

Momentendiagramm



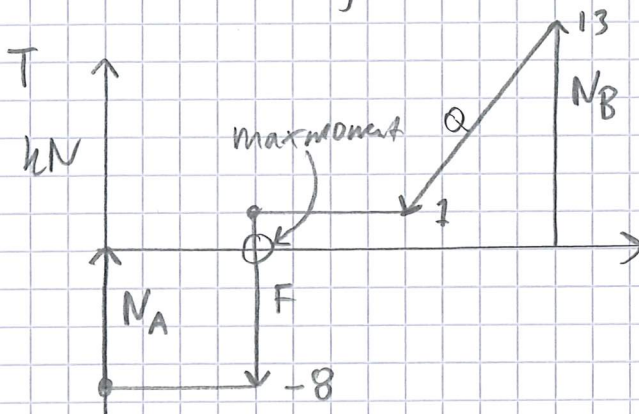
②



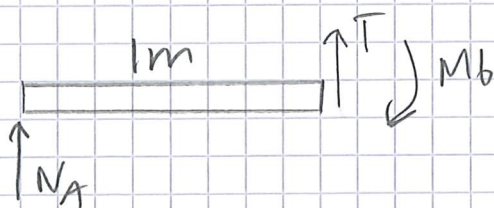
$$\begin{cases} \uparrow: N_A + N_B - F - Q = 0 \\ \curvearrowright: F \cdot 1 - N_B \cdot 3 + Q(3 - 0.5) = 0 \end{cases}$$

$$\Rightarrow N_A = 8 \text{ kN} \text{ och } N_B = 13 \text{ kN}$$

Trätkraftsdiagram



största momentet är

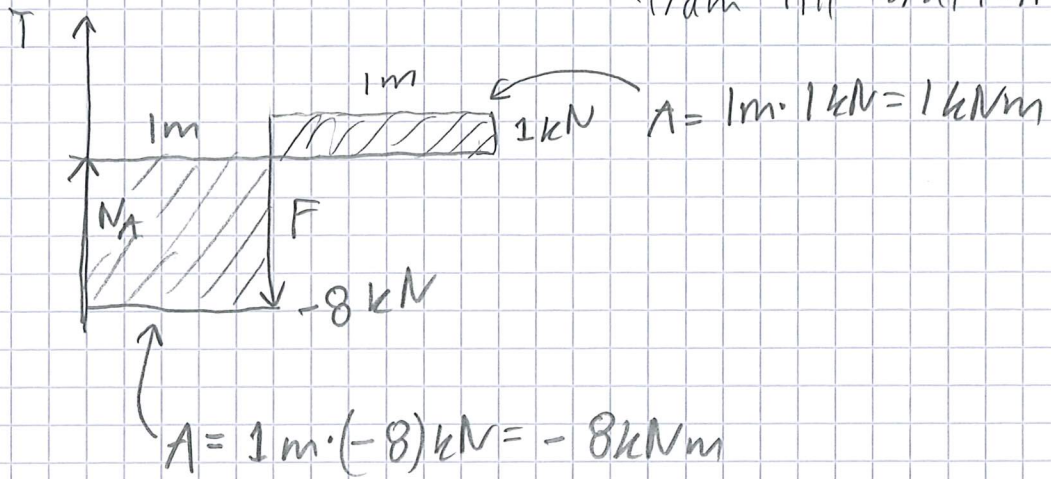


$$\begin{aligned} \curvearrowright: N_A \cdot 1 + M_b &= 0 \\ \Rightarrow M_b &= -N_A \cdot 1 = -8 \cdot 1 = \\ &= \underline{\underline{-8 \text{ kNm}}} \end{aligned}$$

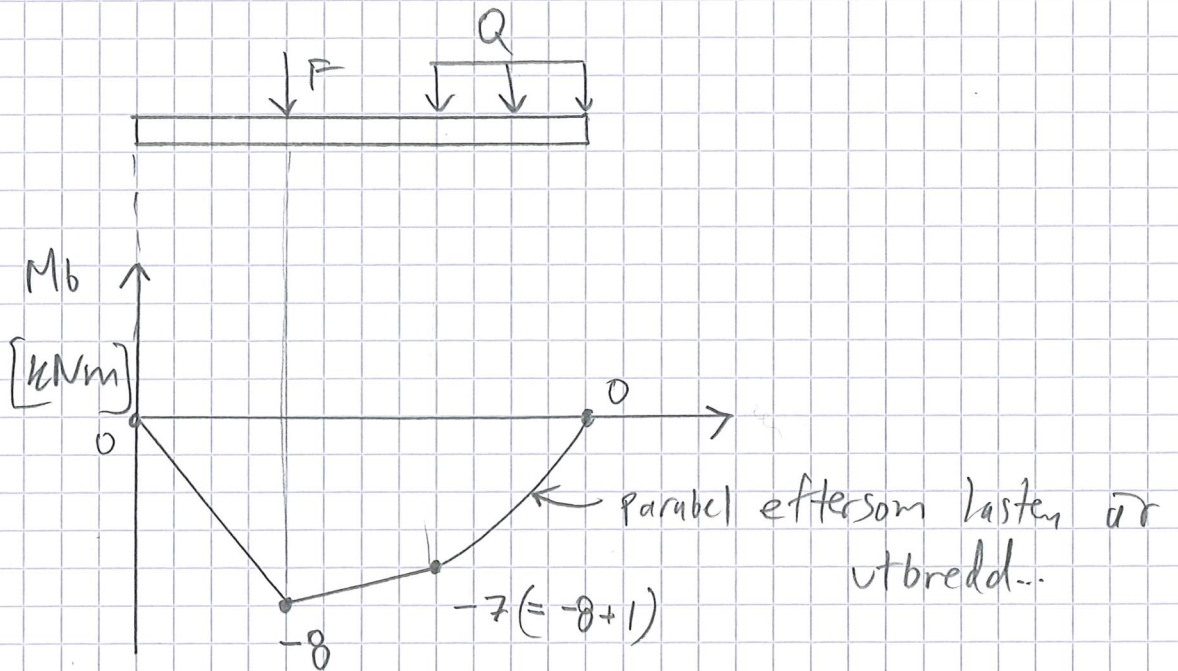
$\boxed{1/2}$

②

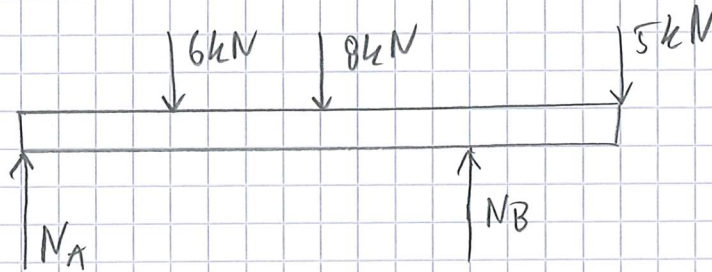
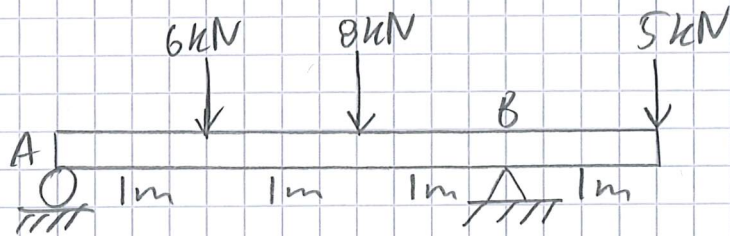
Alt. beräkna arean under T-kurvan fram till kraften F



2.7a momentdiagram

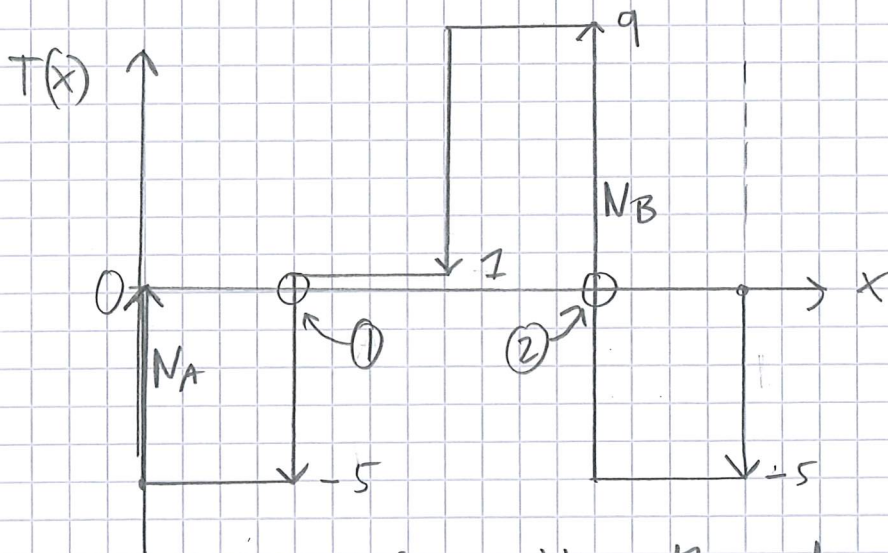


③



$$\begin{cases} \uparrow: N_A + N_B - 6 - 8 - 5 = 0 \\ \rightarrow: 6 \cdot 1 + 8 \cdot 2 - N_B \cdot 3 + 5 \cdot 4 = 0 \end{cases}$$

$$\Rightarrow N_A = 5 \text{ kN} \text{ och } N_B = 14 \text{ kN}$$

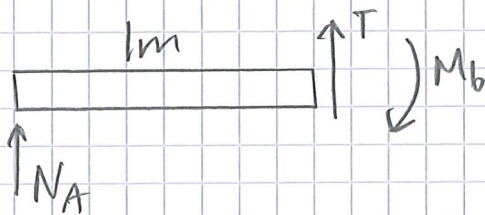


TVå punkter att undersöka

$\frac{1}{2}$

③

Punkt 1 (vid $x=1\text{m}$)

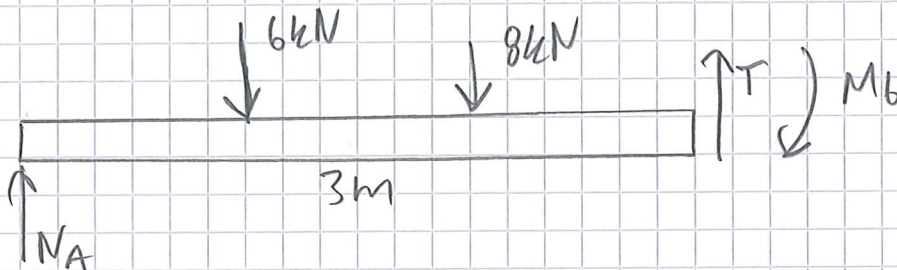


$$\uparrow: N_A + T = 0$$

$$\curvearrowright: N_A \cdot 1 + M_b = 0$$

$$\Rightarrow M_b = -N_A \cdot 1 = -5 \cdot 1 = \underline{\underline{-5 \text{ kNm}}}$$

Punkt 2 (vid $x=3\text{m}$)



$$\uparrow: N_A - 6 - 8 + T = 0$$

$$\curvearrowright: N_A \cdot 3 - 6 \cdot 2 - 8 \cdot 1 + M_b = 0$$

$$5 \cdot 3 - 6 \cdot 2 - 8 + M_b = 0$$

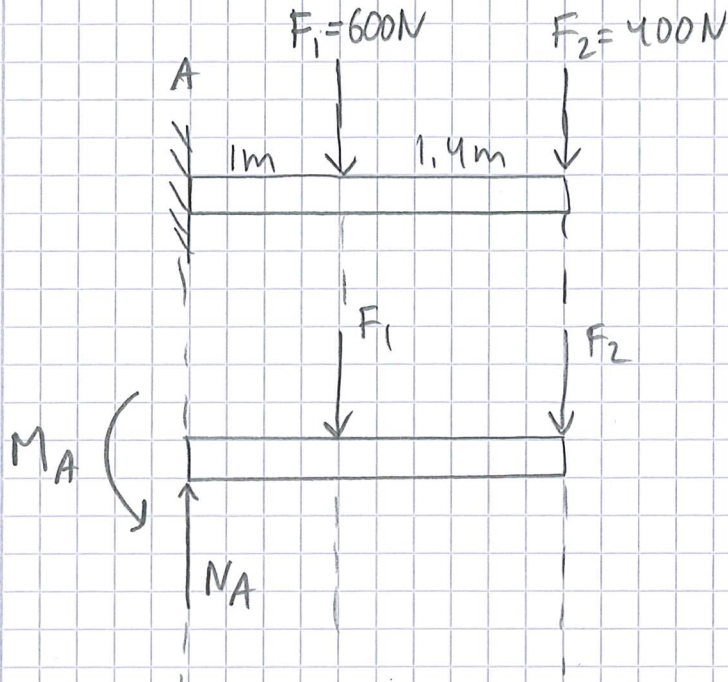
$$15 - 12 - 8 + M_b = 0$$

$$-5 + M_b = 0$$

$$\Rightarrow M_b = \underline{\underline{5 \text{ kNm}}}$$

Största momentet
är 5 kNm vid
 $x=1$ och $x=3$.

(4)

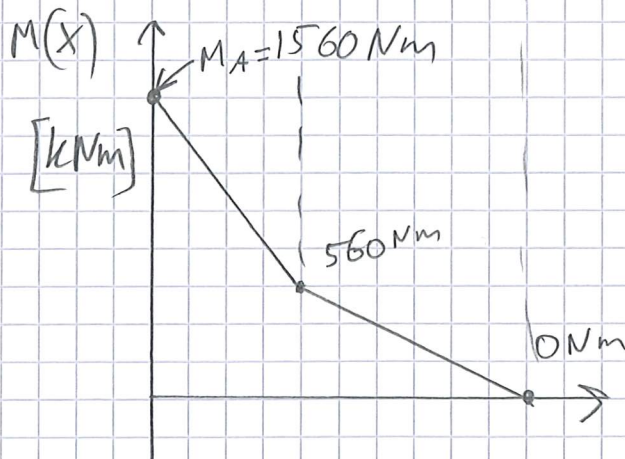
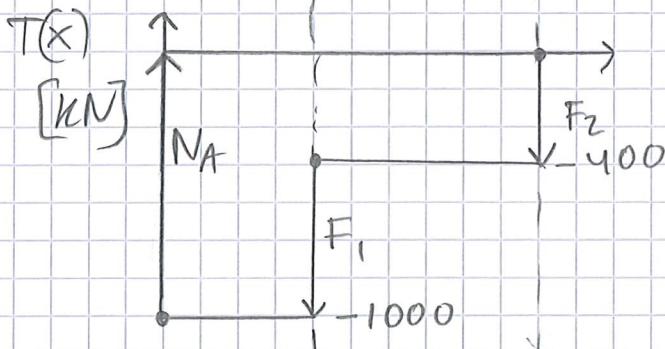


$$\uparrow: N_A - F_1 - F_2 = 0 \Rightarrow N_A = F_1 + F_2 = 600 + 400 = 1000 \text{ N}$$

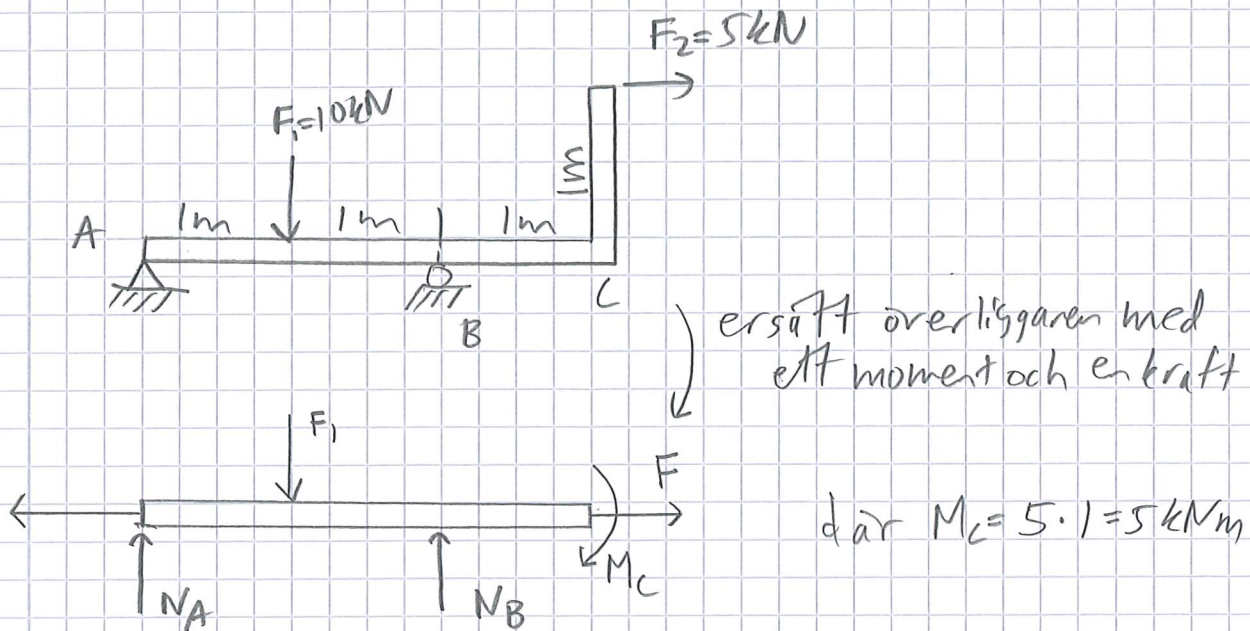
$$\curvearrowleft: F_1 \cdot 1 + F_2 \cdot 2,4 - M_A = 0$$

$$\Rightarrow M_A = F_1 \cdot 1 + F_2 \cdot 2,4 = 600 \cdot 1 + 400 \cdot 2,4 =$$

$$= 1560 \text{ Nm}$$

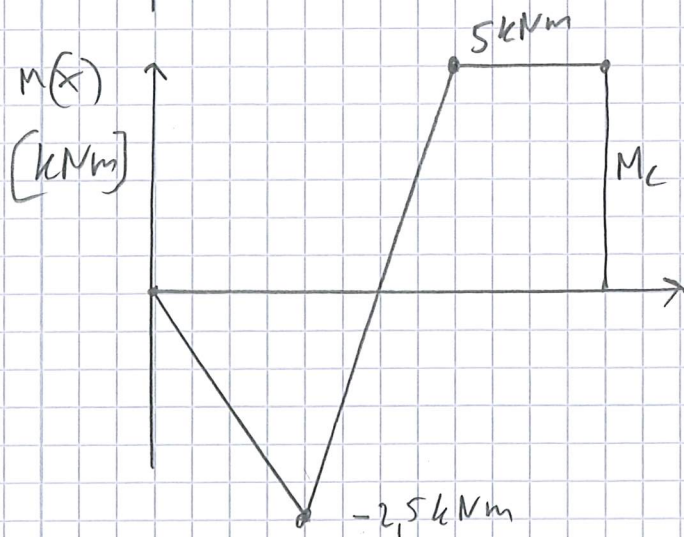
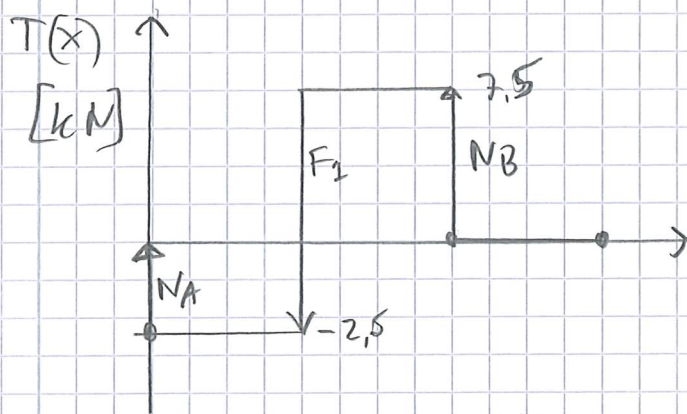


5

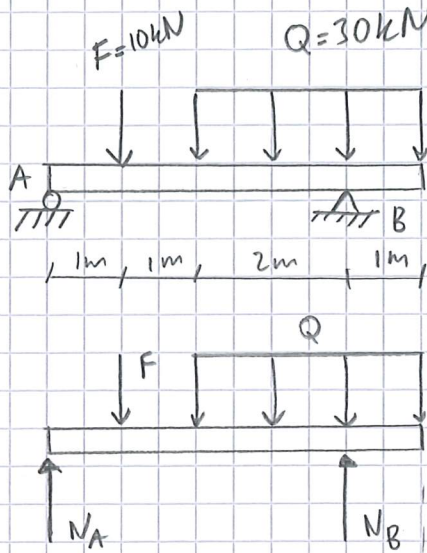


$$\begin{cases} \uparrow: N_A + N_B - F_1 = 0 \\ \curvearrowright: F_1 \cdot 1 - N_B \cdot 2 + M_C = 0 \end{cases}$$

$$\Rightarrow N_A = 2,5\text{ kN} \text{ och } N_B = 7,5\text{ kN}$$

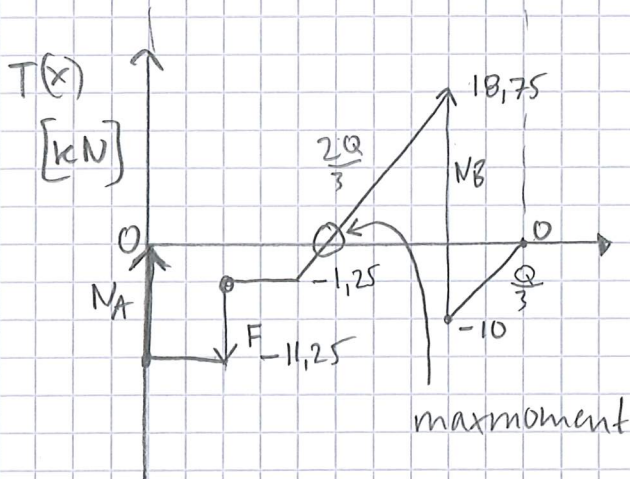


6)



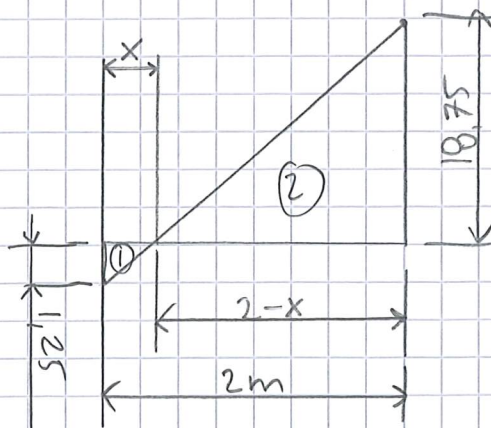
$$\begin{cases} \uparrow: N_A + N_B - F - Q = 0 \\ \curvearrowright: F \cdot 1 + Q \cdot 3,5 - N_B \cdot 4 = 0 \end{cases}$$

$$\Rightarrow N_A = 11,25 \text{ kN} \text{ och } N_B = 28,75 \text{ kN}$$



Var finns maxmomentet?

liksidiga
trianglar
① och ②



$$\frac{2-x}{18,75} = \frac{x}{11,25}$$

$\frac{1}{3}$

6

$$\frac{2-x}{18,75} = \frac{x}{1,25}$$

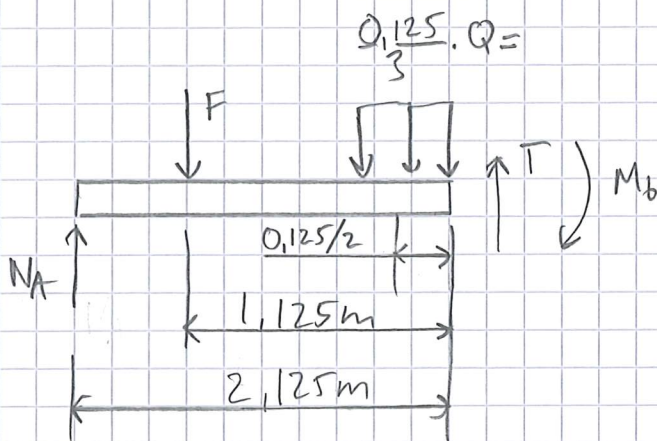
$$\frac{2}{18,75} - \frac{x}{18,75} = \frac{x}{1,25}$$

$$\frac{2}{18,75} = \frac{x}{1,25} + \frac{x}{18,75}$$

$$\frac{2}{18,75} = x \left(\frac{1}{1,25} + \frac{1}{18,75} \right)$$

$$\Rightarrow x = \frac{2}{18,75 \cdot \left(\frac{1}{1,25} + \frac{1}{18,75} \right)} = 0,125 \text{ m}$$

snitt vid $x = 2,125 \text{ m}$



$$\uparrow \curvearrowright: N_A \cdot 2,125 - F \cdot 1,125 - \frac{0,125}{3} \cdot Q \cdot \frac{0,125}{2} + M_b = 0$$

$$M_b = \frac{0,125}{3} \cdot Q \cdot \frac{0,125}{2} + 1,125 F - 2,125 \cdot N_A$$

$$M_b = \frac{0,125 \cdot 30}{3} \cdot \frac{0,125}{2} + 1,125 \cdot 10 - 2,125 \cdot 11,25 = -12,58 \text{ kNm}$$

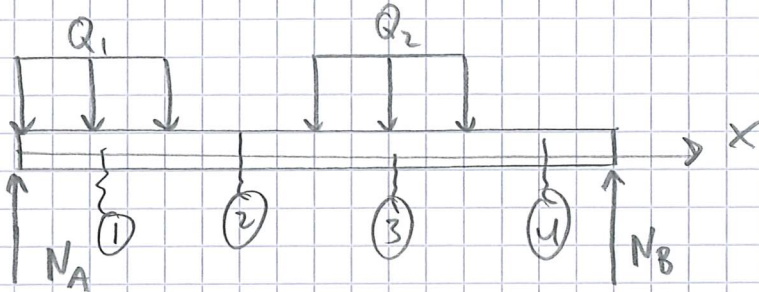
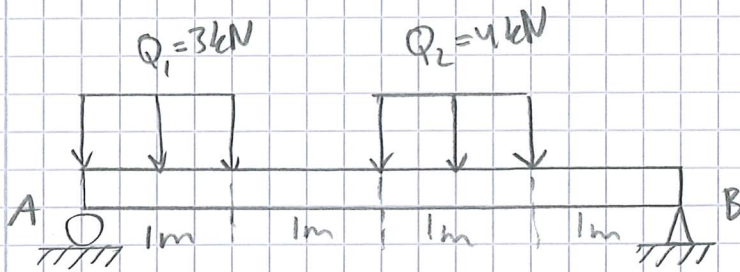
$\frac{2}{3}$

6

$$\sigma_b = \frac{M_b}{W_b} = \frac{-12\,580\,000}{121\,000} \approx 104 \text{ MPa}$$

$$\mu_s = \frac{R_e}{\sigma_b} = \frac{260}{104} = 2,5$$

7



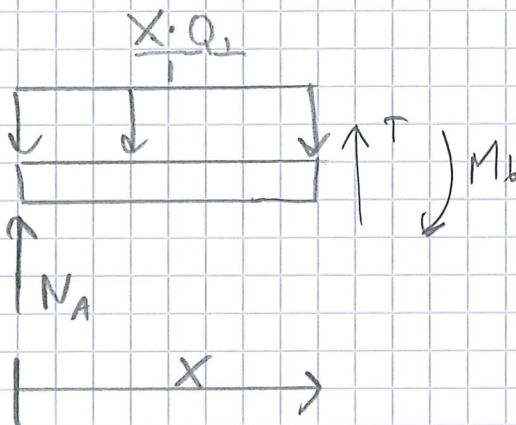
fyra olika snitt ① ② ③ ④

$$\uparrow: N_A + N_B - Q_1 - Q_2 = 0$$

$$\curvearrowright: Q_1 \cdot 0,5 + Q_2 \cdot 2,5 - N_B \cdot 4 = 0$$

$$\Rightarrow N_A = 4,125 \text{ kN}, N_B = 2,875 \text{ kN}$$

Snitt 1, $0 \leq x \leq 1$ (m)



$$\uparrow: N_A + T - x \cdot Q_1 = 0 \Rightarrow T(x) = x \cdot Q_1 - N_A = 3 \cdot x - 4,125 \text{ (kN)}$$

$$\curvearrowright: N_A \cdot x - \frac{x \cdot Q_1 \cdot x}{2} + M_b = 0 \Rightarrow M_b(x) = \frac{x^2 \cdot Q_1}{2} - N_A \cdot x =$$

$$= \frac{3}{2} x^2 - 4,125 x \text{ (kNm)}$$

4/7

7

Test av gränser

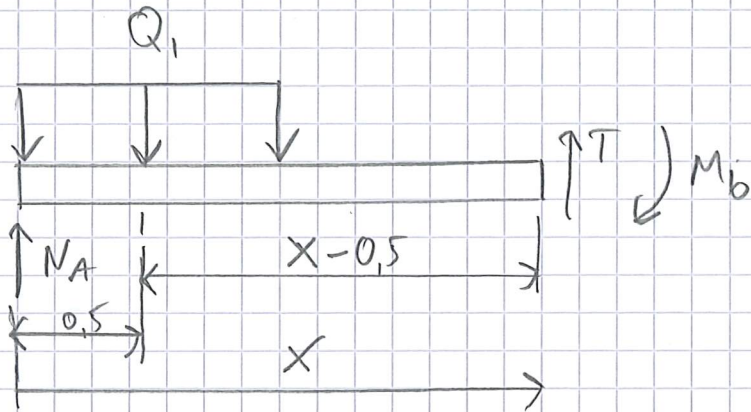
$$T(0) = 3 \cdot 0 - 4,125 = -4,125 \text{ kN}$$

$$T(1) = 3 \cdot 1 - 4,125 = -1,125 \text{ kN}$$

$$M(0) = \frac{3}{2} \cdot 0^2 - 4,125 \cdot 0 = 0 \text{ kNm}$$

$$M(1) = \frac{3}{2} \cdot 1^2 - 4,125 \cdot 1 = -2,625 \text{ kNm}$$

snitt 2, $1 \leq x \leq 2$ (m)



$$\uparrow T: N_A + T - Q_1 = 0 \Rightarrow T(x) = Q_1 - N_A = 3 - 4,125 = -1,125 \text{ kN}$$

$$\curvearrow T: N_A \cdot x - Q_1 \cdot (x - 0,5) + M_b = 0$$

$$M_b(x) = Q_1 \cdot (x - 0,5) - N_A \cdot x = 3x - 1,5 - 4,125 \cdot x = -1,125x - 1,5 \text{ (kNm)}$$

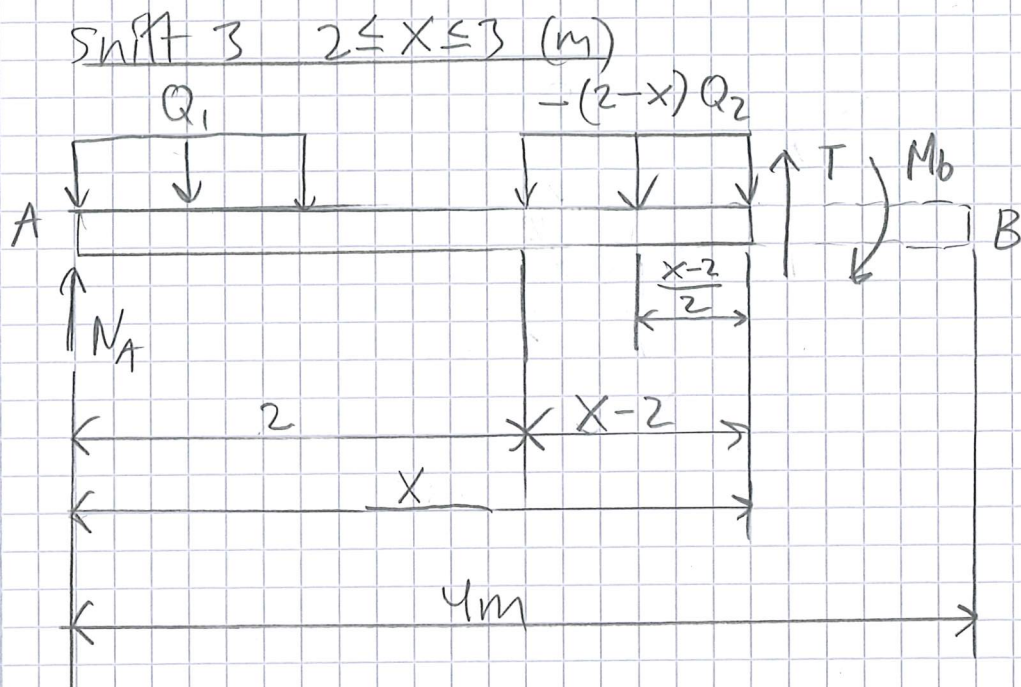
Test av gränser

$$\left. \begin{array}{l} T(1) = -1,125 \text{ kN} \\ T(2) = -1,125 \text{ kN} \end{array} \right\} \text{konstant}$$

$$M(1) = -1,125 \cdot 1 - 1,5 = -2,625 \text{ kNm}$$

$$M(2) = -1,125 \cdot 2 - 1,5 = -3,75 \text{ kNm}$$

7



Hur stor del av Q_2 får vi med?

Om $x=2 \Rightarrow Q_2=0$ (ingen last är med)

Om $x=3 \Rightarrow Q_2=Q_2$ (hela lasten är med)

$$\Rightarrow -\left(\frac{2-x}{1}\right) Q_2$$

Test

$$\text{Om } x=2 \Rightarrow -(2-2)Q_2=0 \text{ kN}$$

$$\text{Om } x=3 \Rightarrow -(2-3)Q_2=1 \cdot Q_2=Q_2 \text{ kN}$$

$$\uparrow: N_A - Q_1 - [-(2-x)Q_2] + T = 0$$

$$\Rightarrow T(x) = Q_1 + [-(2-x)Q_2] - N_A =$$

$$= 3 + [-2Q_2 + xQ_2] - 4,125 =$$

$$= 3 - 8 + x \cdot 4 - 4,125 =$$

$$= -9,125 + 4x \text{ (kN)}$$

$\boxed{\frac{3}{7}}$

$$\textcircled{7} \quad \uparrow: N_A \cdot X - Q_1 \cdot (X - 0,5) - [-(2-x)Q_2] \cdot \frac{X-2}{2} + M_b = 0$$

$$\Rightarrow 4,125 \cdot X - 3(X - 0,5) - [-(2-x)4] \cdot \frac{X-2}{2} + M_b = 0$$

$$\Rightarrow 4,125X - 3X + 1,5 - [-8 + 4x] \cdot \frac{X-2}{2} + M_b = 0$$

$$\Rightarrow 4,125X - 3X + 1,5 + 8\left(\frac{X-2}{2}\right) - 4x\left(\frac{X-2}{2}\right) + M_b = 0$$

$$1,125X + 1,5 + \frac{8X}{2} - \frac{16}{2} - \frac{4X^2}{2} + \frac{8X}{2} + M_b = 0$$

$$X(1,125 + 4 + 4) + 1,5 - 8 - 2X^2 + M_b = 0$$

$$9,125X - 6,5 - 2X^2 + M_b = 0$$

$$M_b(x) = 2x^2 - 9,125x + 6,5 \text{ (kNm)}$$

Test an gränser

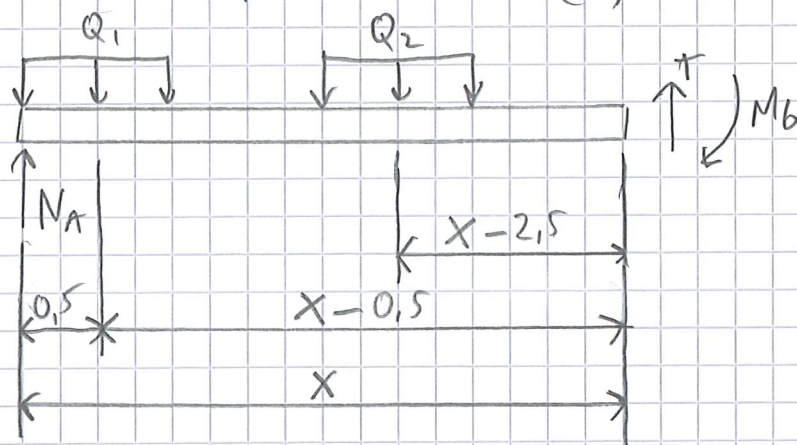
$$T(2) = -9,125 + 4 \cdot 2 = -1,125 \text{ kN}$$

$$T(3) = -9,125 + 4 \cdot 3 = 2,875 \text{ kN}$$

$$M(2) = 2 \cdot 2^2 - 9,125 \cdot 2 + 6,5 = -3,75 \text{ kNm}$$

$$M(3) = 2 \cdot 3^2 - 9,125 \cdot 3 + 6,5 = -2,875 \text{ kNm}$$

Snitt 4, $3 \leq x \leq 4$ (m)



$$\begin{aligned} \uparrow: N_A - Q_1 - Q_2 + T &= 0 \Rightarrow T(x) = Q_1 + Q_2 - N_A = \\ &= 3 + 4 - 4,125 = 2,875 \\ &= 2,875 \text{ kN} \end{aligned}$$

$\frac{4}{7}$

⑦

$$\uparrow: N_A \cdot x - Q_1 \cdot (x - 0,5) - Q_2 \cdot (x - 2,5) + M_b = 0$$

$$4,125 \cdot x - 3(x - 0,5) - 4(x - 2,5) + M_b = 0$$

$$4,125x - 3x + 1,5 - 4x + 10 + M_b = 0$$

$$x(4,125 - 3 - 4) + 1,5 + 10 + M_b = 0$$

$$-2,875x + 11,5 + M_b = 0$$

$$M_b(x) = 2,875x - 11,5$$

Test av gränser

$$T(3) = 2,875 \text{ kN} \left. \vphantom{T(3)} \right\} \text{konstant}$$

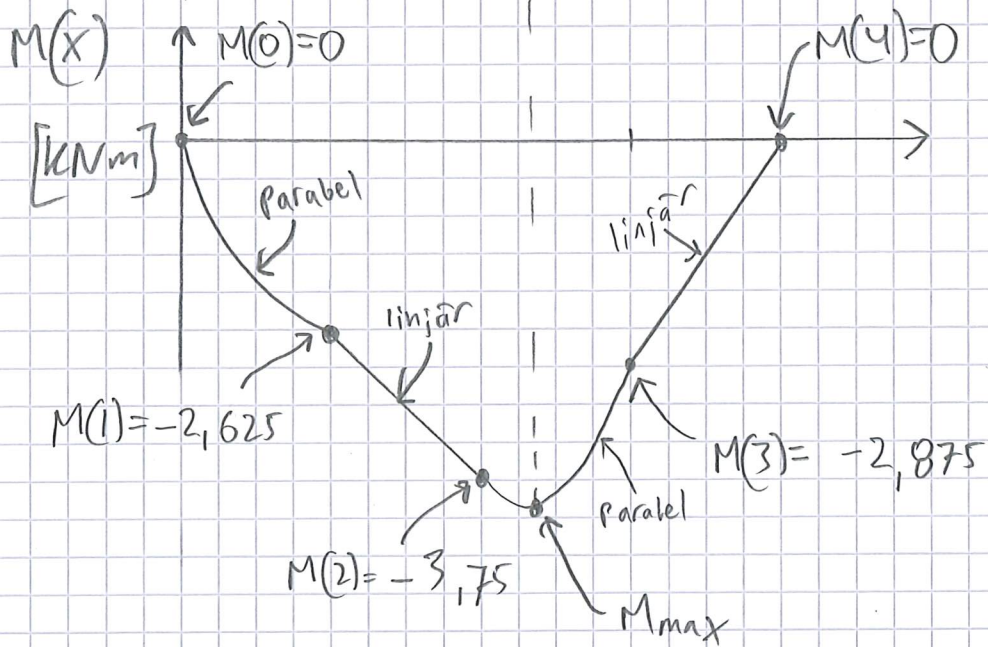
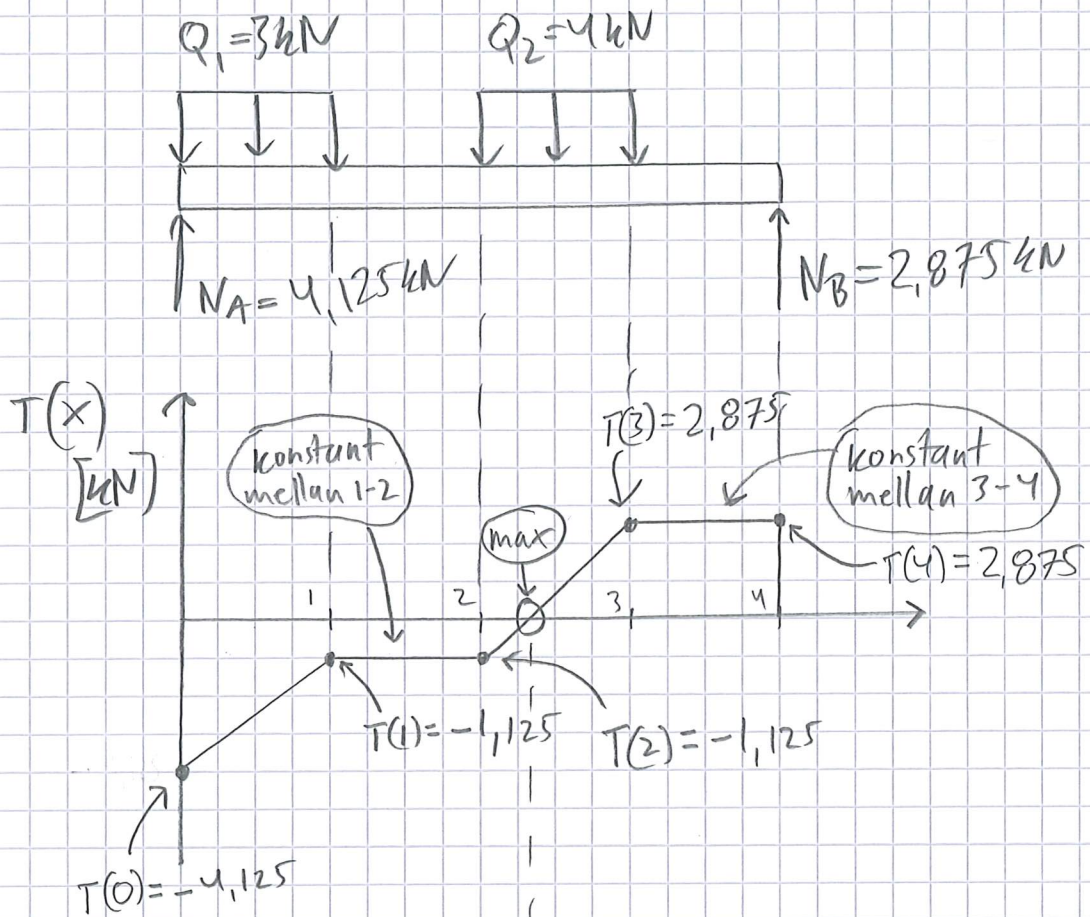
$$T(4) = 2,875 \text{ kN} \left. \vphantom{T(4)} \right\}$$

$$M(3) = 2,875 \cdot 3 - 11,5 = -2,875 \text{ kNm}$$

$$M(4) = 2,875 \cdot 4 - 11,5 = 0 \text{ kNm}$$

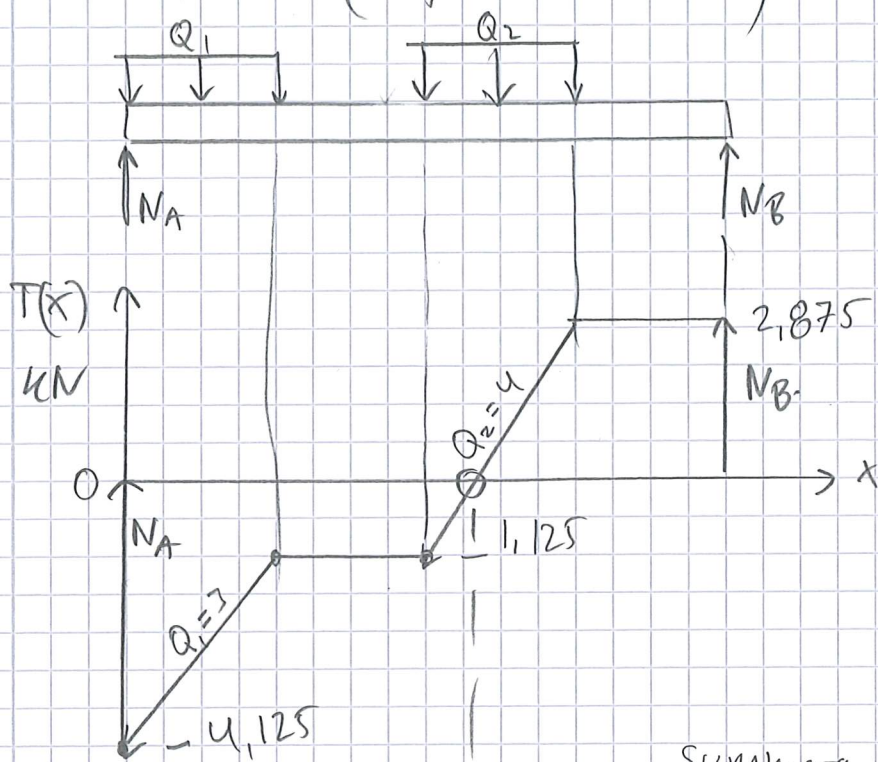
Nu finns det funktioner för
att rita T- och M-diagram för
hela balken

7

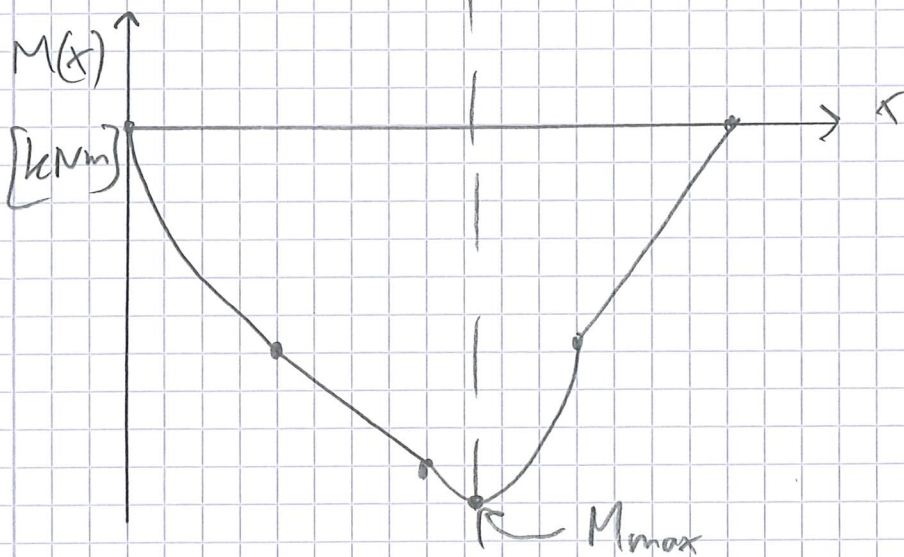


7

Kontroll (börja från höger)

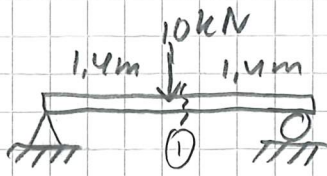


Summera area under $T(x)$
börja från vänster



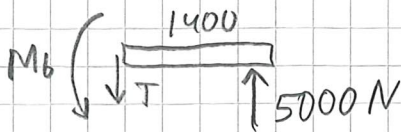
Ok, stämmer

⑧



$$n_s = 2,5, \quad R_e = 260 \text{ MPa}$$

Snitta vid ①



$$\overset{\curvearrowright}{T}: -5000 \cdot 1400 - M_b = 0 \quad \Rightarrow \quad M_b = -7000000 \text{ Nmm}$$

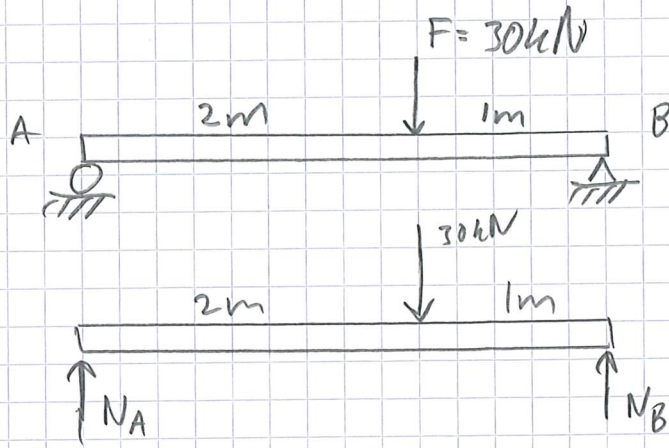
$$\sigma_{b, till} = \frac{R_e}{n_s} = \frac{260}{2,5} = 104 \text{ MPa}$$

$$\sigma_b = \frac{M_b}{W_b} \Rightarrow \quad W_b = \frac{M_b}{\sigma_{b, till}} = -\frac{7000000}{104} \approx 67308 \text{ mm}^3 =$$

$$= 67 \text{ cm}^3$$

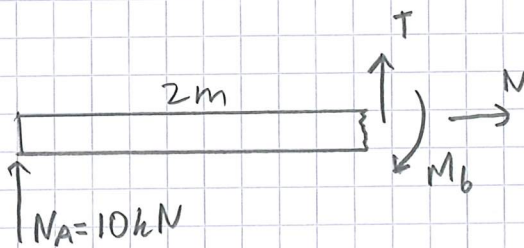
Välj I140 som har $W_x = 81,9 \text{ cm}^3$ (stående)

9



$$\left\{ \begin{array}{l} \uparrow: N_A + N_B - 30 = 0 \quad (\text{kN}) \\ \rightarrow: 0 \\ \curvearrowright: 30 \cdot 2 - N_B \cdot 3 = 0 \quad (\text{kNm}) \end{array} \right.$$

$$\Rightarrow N_B = 20 \text{ kN} \quad \text{och} \quad N_A = 10 \text{ kN}$$



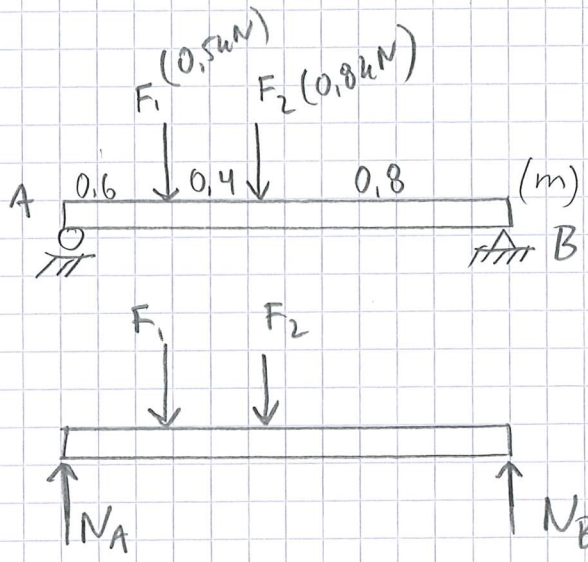
$$\curvearrowright: N_A \cdot 2 + M_b = 0$$

$$\Rightarrow M_b = -N_A \cdot 2 = -10 \cdot 2 = -20 \text{ kNm}$$

$$\sigma_b = \frac{M_b}{W_b} = \frac{-20\,000\,000}{172\,000} \approx 116,3 \text{ MPa}$$

$$n_s = \frac{R_e}{\sigma_b} = \frac{290}{116,3} \approx 2,5$$

10



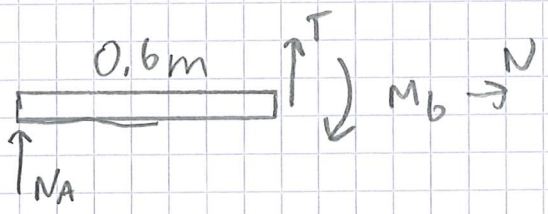
$R_c = ?$
 $n_s = 5,4$
 $w_b = 10,2 \text{ cm}^3$

$$\begin{cases} \uparrow: N_A + N_B - F_1 - F_2 = 0 \\ \curvearrowright: F_1 \cdot 0,6 + F_2 \cdot 1,0 - N_B \cdot 1,8 = 0 \end{cases}$$

$$\Rightarrow N_A = 689 \text{ N} \quad \text{och} \quad N_B = 611 \text{ N}$$

Var är största momentet?

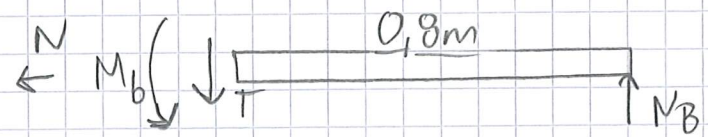
Snitt ett vid F_1



$$\curvearrowright: N_A \cdot 0,6 + M_b = 0$$

$$\Rightarrow M_b = -N_A \cdot 0,6 = -689 \cdot 0,6 = -413,4 \text{ Nm}$$

Snitt två vid F_2



$$\curvearrowright: -N_B \cdot 0,8 - M_b = 0 \quad \Rightarrow \quad M_b = -N_B \cdot 0,8 = -611 \cdot 0,8 = -488,8 \text{ Nm}$$

1/2

10

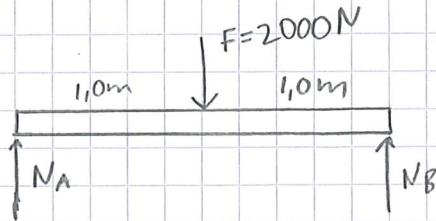
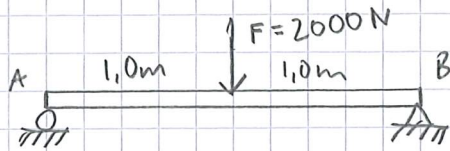
Största momentet är $\{-488,8 \text{ Nm}\}$

$$\sigma_{\max} = \frac{M_b}{W_b} = -\frac{488\,800}{10\,200} \approx 47,92 \text{ MPa}$$

$$n_s = \frac{R_e}{\sigma_{\max}} \Rightarrow R_e = n_s \cdot \sigma_{\max} = 5,4 \cdot 47,92 \approx 258,8 \text{ MPa}$$

$\frac{2}{2}$

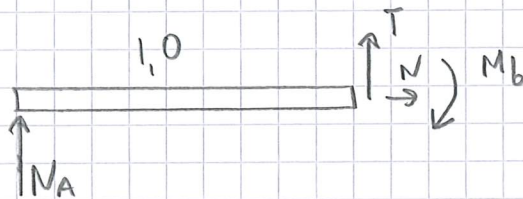
11



$$\begin{cases} \uparrow: N_A + N_B - F = 0 \\ \curvearrowright: F \cdot 1,0 - N_B \cdot 2,0 = 0 \end{cases}$$

$$\Rightarrow N_A = 1000\text{N} \text{ och } N_B = 1000\text{N}$$

Största momentet är



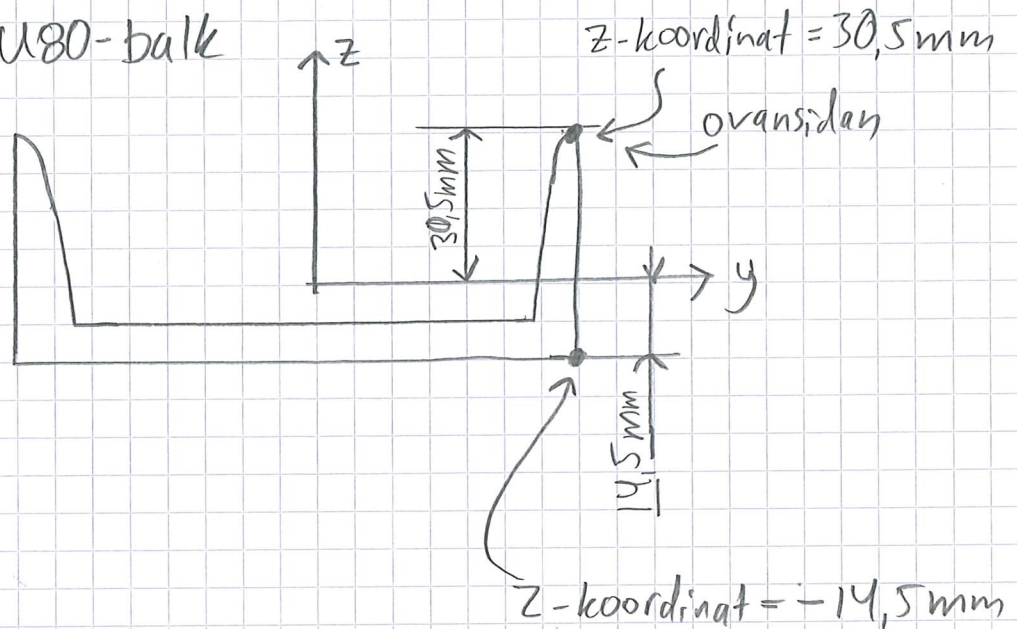
$$\curvearrowright: N_A \cdot 1,0 + M_b = 0$$

$$\Rightarrow M_b = -N_A \cdot 1,0 = -1000 \cdot 1,0 = -1000\text{ Nm}$$

1/2

(11)

U80-balk



Spänningen på ovansidan

$$\sigma_b = \frac{M_b \cdot e_{max}}{I} \quad \text{där } e_{max} = \text{koordinat längs } z\text{-axeln}$$

$$\sigma_b = \frac{-1000\,000 \cdot 30,5}{194\,000} \approx -157 \text{ MPa} \quad (\text{Tryckspänning})$$

Spänningen på undersidan

$$\sigma_b = \frac{-1000\,000 \cdot (-14,5)}{194\,000} \approx 74,8 \text{ MPa} \quad (\text{Dragspänning})$$

