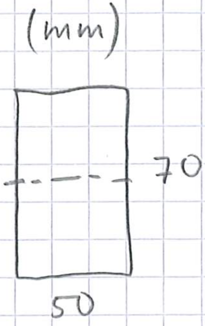
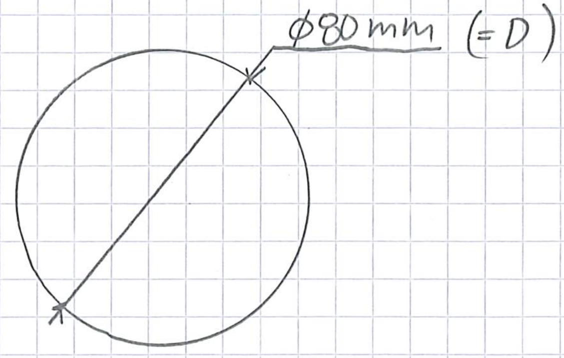
1



$$W_b = \frac{b \cdot h^2}{6} = \frac{50 \cdot 70^2}{6} \approx 40\,833 \text{ mm}^3$$

$$I = \frac{b \cdot h^3}{12} = \frac{50 \cdot 70^3}{12} \approx 1\,429\,167 \text{ mm}^4$$

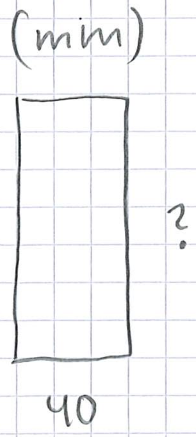
(2)



$$W_b = \frac{\pi D^3}{32} = \frac{\pi \cdot 80^3}{32} \approx 50\,265 \text{ mm}^3$$

$$I = \frac{\pi \cdot D^4}{64} = \frac{\pi \cdot 80^4}{64} \approx 2\,010\,619 \text{ mm}^4$$

3

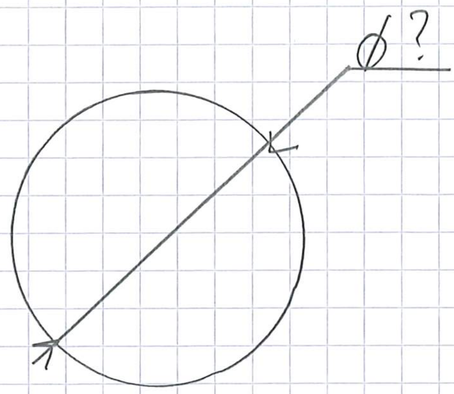


där $b=40$

$$W_b = 40\ cm^3 = 40\,000\ mm^3 = \frac{b \cdot h^2}{6}$$

$$\Rightarrow \quad h = \sqrt{\frac{6 \cdot 40\,000}{40}} \approx 77{,}5\ mm$$

(4)



$W_b = 25\,cm^3 = 25\,000\,mm^3$

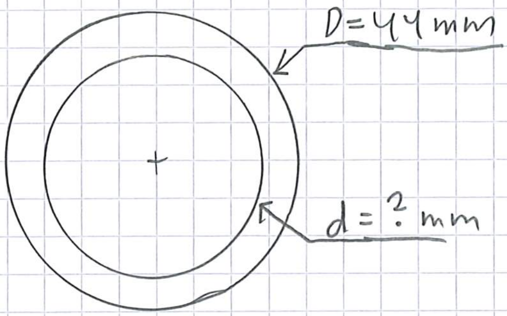
$$W_b = \frac{\pi \cdot D^3}{32}$$

$$D = \left(\frac{32 \cdot W_b}{\pi}\right)^{1/3} = \left(\frac{32 \cdot 25\,000}{\pi}\right)^{1/3} \approx 63{,}4\,mm$$

⑤

$$W_b = \frac{\pi}{32 \cdot D}(D^4 - d^4) = \frac{\pi}{32 \cdot 68}(68^4 - 60^4) \approx 12\,158 \, mm^3$$

6



$$W_b = 4\,000 \text{ mm}^3$$

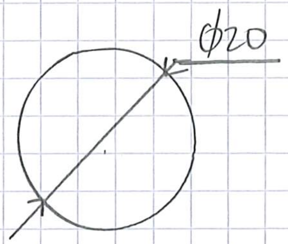
$$W_b = \frac{\pi}{32D}\left(D^4 - d^4\right)$$

$$\frac{32 \cdot D \cdot W_b}{\pi} = D^4 - d^4$$

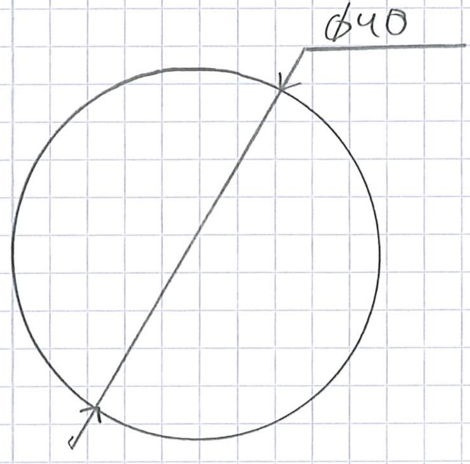
$$d = \left(D^4 - \frac{32D \cdot W_b}{\pi}\right)^{1/4} = \left(44^4 - \frac{32 \cdot 44 \cdot 4000}{\pi}\right)^{1/4} \approx$$

$$\approx 37{,}4 \text{ mm}$$

(7)



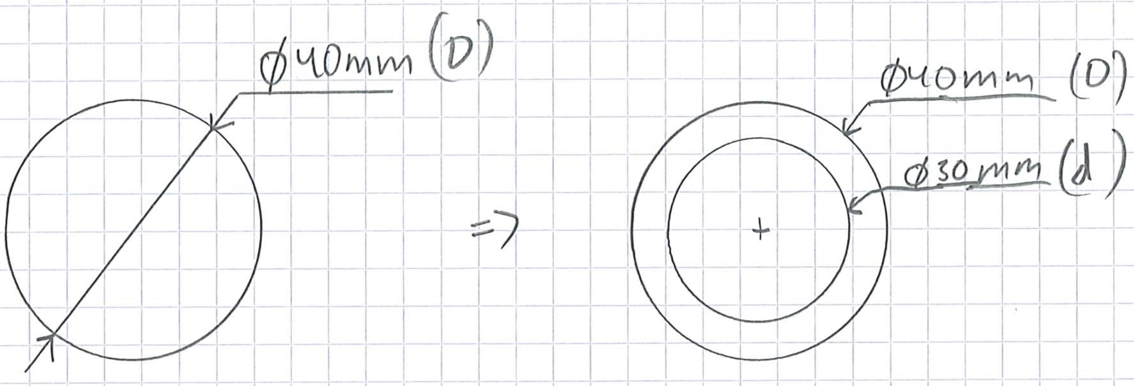
$$W_{b_1} = \frac{\pi D^3}{32} = \frac{\pi \cdot 20^3}{32}$$



$$W_{b_2} = \frac{\pi D^3}{32} = \frac{\pi \cdot 40^3}{32}$$

$$\text{skillnad} = \frac{W_{b_2}}{W_{b_1}} = \frac{\frac{\pi\, 40^3}{32}}{\frac{\pi \cdot 20^3}{32}} = \frac{40^3}{20^3} = \underline{\underline{8}}$$

8



$$W_{b_s} = \frac{\pi D^3}{32} = \frac{\pi \cdot 40^3}{32}\ \text{mm}^3 \qquad W_{b_r} = \frac{\pi}{32D}\left(D^4 - d^4\right) = \frac{\pi}{32\cdot 40}\left(40^4 - 30^4\right)$$

Minskningen i procent

$$100\cdot\left(1 - \frac{W_{br}}{W_{bs}}\right) = 100\left(1 - \frac{\frac{\pi}{32\cdot 40}\left(40^4-30^4\right)}{\frac{\pi\cdot 40^3}{32}}\right) \approx 31{,}6\,\%$$

Vikt per meter

Stång: $m_s = \rho\cdot V = \rho\cdot A\cdot L = \rho\cdot\frac{\pi D^2}{4}\cdot L = 7800\cdot\frac{\pi\cdot 0{,}04^2}{4}\cdot 1\ \text{kg}$

Rör: $m_r = \rho\cdot A\cdot L = \rho\left(\frac{\pi}{4}\left(D^2 - d^2\right)\right)\cdot L = 7800\cdot\frac{\pi}{4}\left(0{,}04^2 - 0{,}03^2\right) 1\ \text{kg}$

Röret är hur många procent lättare?

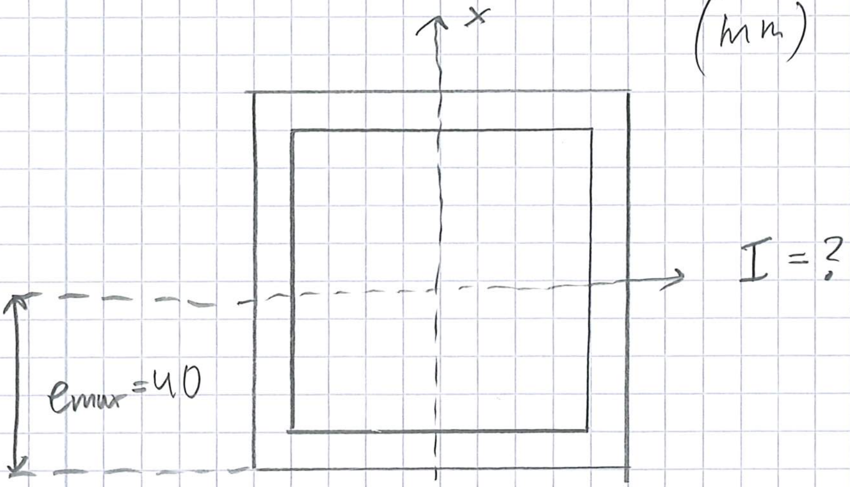
$$100\left(1 - \frac{m_r}{m_s}\right) = 100\cdot\left(1 - \frac{7800\cdot\frac{\pi}{4}\left(0{,}04^2 - 0{,}03^2\right)\cdot 1}{7800\cdot\frac{\pi}{4}\ 0{,}04^2\cdot 1}\right) \approx 56\,\%$$

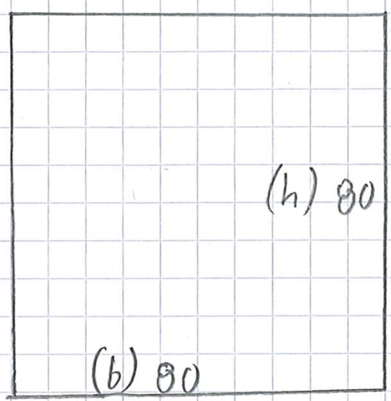
Meritvärde

$$\frac{W_{bs}}{m_s} = \frac{\frac{\pi\cdot 40^3}{32}}{7800\cdot\frac{\pi}{4}0{,}04^2\cdot 1} \approx 641\ \frac{\text{mm}^3}{\text{kg}}$$

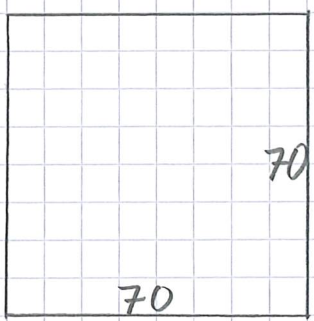
$$\frac{W_{br}}{m_r} = \frac{\frac{\pi}{32\cdot 40}\left(40^4 - 30^4\right)}{7800\cdot\frac{\pi}{4}\left(0{,}04^2 - 0{,}03^2\right)\cdot 1} \approx 1002\ \frac{\text{mm}^3}{\text{kg}}$$

⑨





$$I_y = \frac{b \cdot h^3}{12} = \frac{80 \cdot 80^3}{12}$$



$$I_i = \frac{b \cdot h^3}{12} = \frac{70 \cdot 70^3}{12}$$

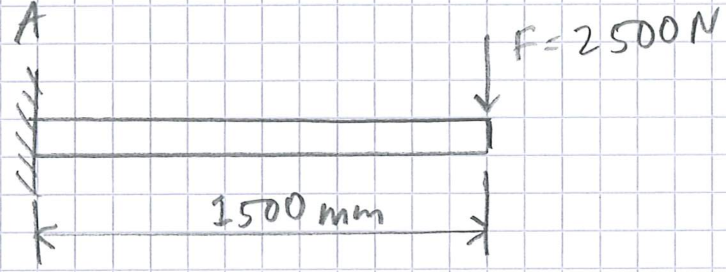
$$I = I_y - I_i = \frac{80 \cdot 80^3}{12} - \frac{70 \cdot 70^3}{12} = 1\ 412\ 500 \text{ mm}^4$$

$$W_b = \frac{I}{e_{max}} = \frac{1\ 412\ 500}{40} = \underline{\underline{35\ 312{,}5 \text{ mm}^3}}$$

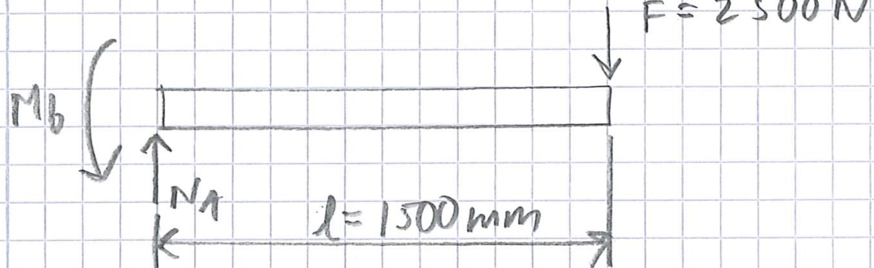
Obs! böjmotståndet kan <u>ej</u> beräknas som

$$\frac{b \cdot h^2}{6} - \frac{b \cdot h^2}{6}$$

10

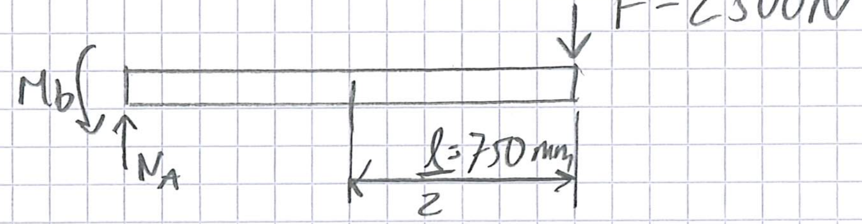


Momentet vid balkens infästning



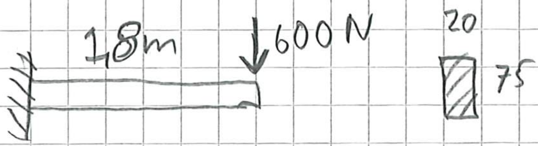
$$M_b = F \cdot l = 2500 \cdot 1500 = 3\,750\,000 \text{ Nmm} =$$
$$= 3{,}75 \text{ kNm}$$

Momentet på mitten av balken



$$M_{b_{mitten}} = F \cdot \frac{l}{2} = 2500 \cdot \frac{1500}{2} = 1\,875\,000 \text{ Nmm} =$$
$$= 1{,}875 \text{ kNm}$$

(11)

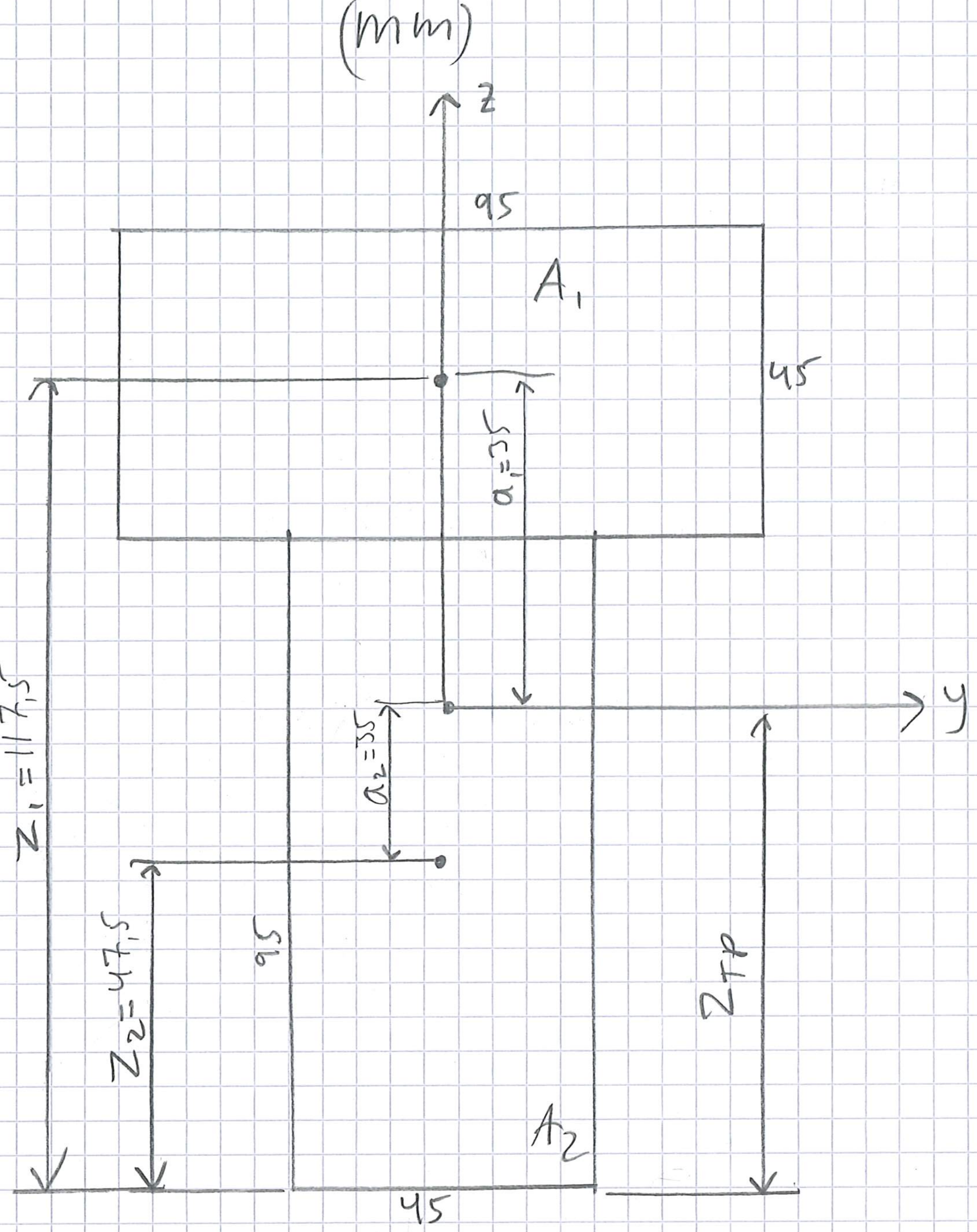


$Re = 220\ MPa$

$$\sigma_b = \frac{M_b}{W_b} \quad \text{där} \quad W_b = \frac{b \cdot h^2}{6} = \frac{20 \cdot 75^2}{6}$$

$$\sigma_b = \frac{600 \cdot 1800}{\frac{20 \cdot 75^2}{6}} \approx \underline{\underline{57{,}6\ MPa}}$$

12



$$z_{TP} = \frac{A_1 \cdot z_1 + A_2 \cdot z_2}{A_1 + A_2} = \frac{(95\cdot 45)\cdot 117{,}5 + (45\cdot 95)\cdot 47{,}5}{(95\cdot 45) + (95\cdot 45)} = 82{,}5 \text{ mm}$$

$\boxed{1/2}$

(12)

$$I_y = I_1 + A_1 \cdot a_1^2 + I_2 + A_2 \cdot a_2^2 =$$

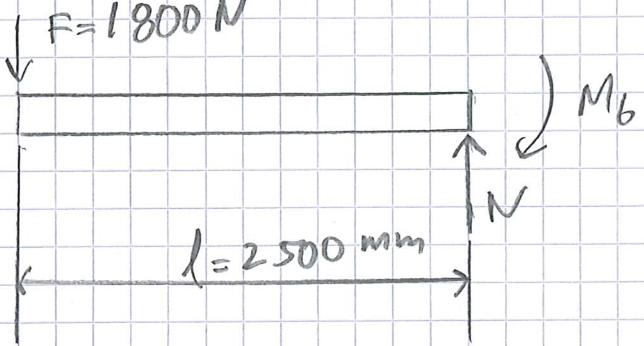
$$= \frac{95 \cdot 45^3}{12} + (95 \cdot 45) \cdot 35^2 + \frac{45 \cdot 95^3}{12} + (45 \cdot 95) \cdot 35^2 =$$

$$= 14\ 410\ 312{,}5\ mm^4 \quad (\approx 1441\ cm^4)$$

$$W_{by} = \frac{I_y}{e_{max}} = \frac{14\ 410\ 312{,}5}{82{,}5} \approx 174\ 670{,}5\ mm^3$$

$$(\approx 175\ cm^3)$$

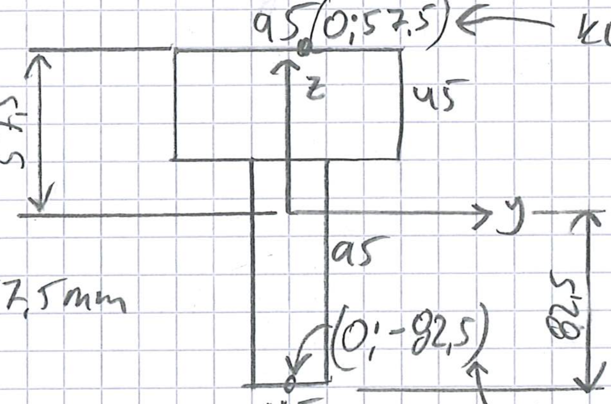
(13) Balken friläggs



Största momentet är Mb:

$$M_b = F \cdot d = 1800 \cdot 2500 = 4\,500\,000 \text{ Nmm}$$

Tvärsnittet ritas upp



$e_{ovan} = 57{,}5$ mm

$I_y = 14\,410\,312{,}5 \text{ mm}^4$

$e_{under} = 82{,}5$ mm

Spänning på ovansidan:

$$\sigma_b = \frac{M_b \cdot e_{ovan}}{I} = \frac{4\,500\,000 \cdot 57{,}5}{14\,410\,312{,}5} \approx 18{,}3 \text{ MPa}$$

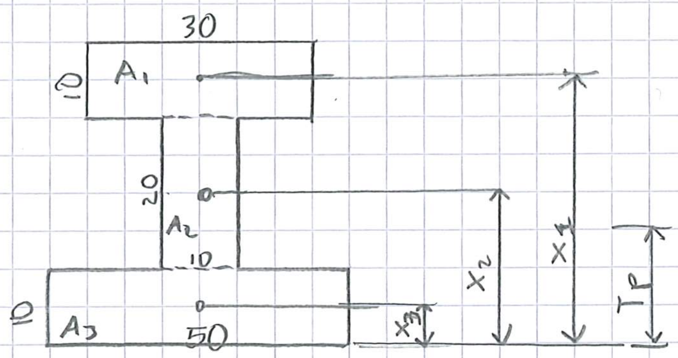
dragspänning

spänning på undersidan:

$$\sigma_b = \frac{M_b \cdot e_{under}}{I} = \frac{4\,500\,000 \cdot (-82{,}5)}{14\,410\,312{,}5} \approx -26{,}3 \text{ MPa}$$

tryckspänning

(14)

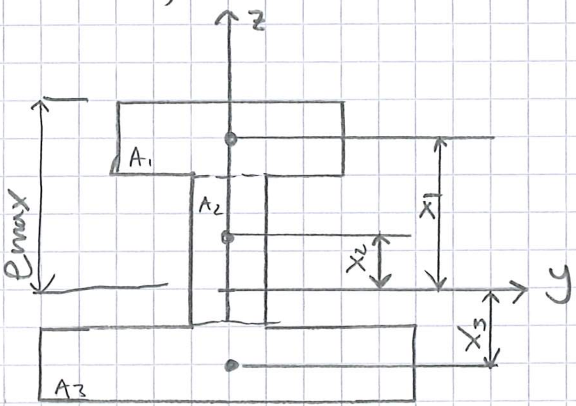


Tyngdpunktsläget beräknas

$$Tp = \frac{A_3 \cdot X_3 + A_2 \cdot X_2 + A_1 \cdot X_1}{A_1 + A_2 + A_3} = \frac{(10\cdot 50)\cdot 5 + (10\cdot 20)\cdot 20 + (10\cdot 30)\cdot 35}{(10\cdot 50) + (10\cdot 20) + (10\cdot 30)} =$$

$$= \underline{\underline{17\ mm}}$$

Yttröghetsmomentet



$X_1 = 35 - 17 = 18\ mm$

$X_2 = 20 - 17 = 3\ mm$

$X_3 = 17 - 5 = 12\ mm$

$$I = \frac{b \cdot h^3}{12}$$

$$I = I_3 + (A_3 \cdot X_3^2) + I_2 + (A_2 \cdot X_2^2) + I_1 + (A_1 \cdot X_1^2) =$$

$$= \frac{50 \cdot 10^3}{12} + (50 \cdot 10 \cdot 12^2) + \frac{10 \cdot 20^3}{12} + (10 \cdot 20 \cdot 3^2) + \frac{30 \cdot 10^3}{12} + (30 \cdot 10 \cdot 18^2) =$$

$\approx 184\ 333\ mm^4$

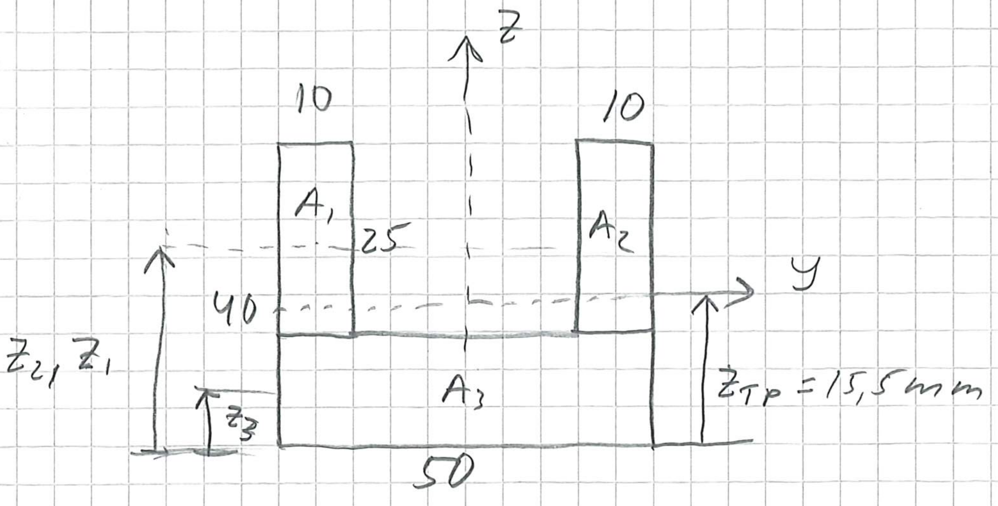
$\approx 18{,}4\ cm^4$

Böjmotståndet

$$W_b = \frac{I}{e_{max}} = \frac{184\ 333}{40 - 17} \approx 8015\ mm^3$$

$$\approx 8\ cm^3$$

(15)



$$I = \frac{b \cdot h^3}{12}$$

Tyngdpunktensläge beräknas

$$z_{TP} = \frac{A_1 \cdot z_1 + A_2 \cdot z_2 + A_3 z_3}{A_1 + A_2 + A_3} =$$

$$= \frac{(10 \cdot 25) \cdot 27{,}5 + (10 \cdot 25) \cdot 27{,}5 + (50 \cdot 15) \cdot 7{,}5}{(10 \cdot 25) + (10 \cdot 25) + (50 \cdot 15)} = 15{,}5\,mm$$

Tröghetsmomentet med steiners sats blir

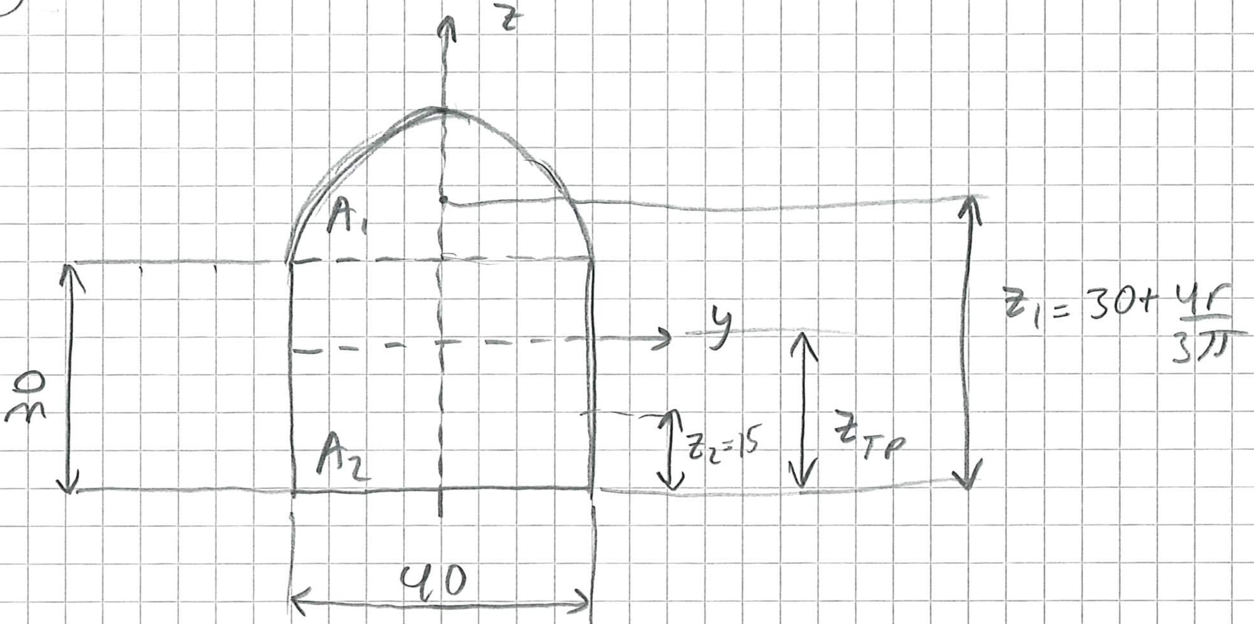
$$I_y = I_1 + A_1 a_1^2 + I_2 + A_2 a_2^2 + I_3 + A_3 a_3^2 =$$

$$= \frac{10 \cdot 25^3}{12} + (10 \cdot 25) \cdot (27{,}5 - 15{,}5)^2 + \frac{10 \cdot 25^3}{12} + (10 \cdot 25) \cdot (27{,}5 - 15{,}5)^2 +$$

$$+ \frac{50 \cdot 15^3}{12} + (50 \cdot 15) \cdot (7{,}5 - 15{,}5)^2 = \underline{\underline{16\ cm^4}}$$

$$W_{by} = \frac{I_y}{e_{max}} = \frac{160104}{24{,}5} \approx \underline{\underline{6{,}54\,cm^3}}$$

16



$$z_{TP} = \frac{A_1 \cdot z_1 + A_2 \cdot z_2}{A_1 + A_2} = \frac{\left(\frac{\pi 40^2}{4 \cdot 2}\right) \cdot \left(30 + \frac{4 \cdot 20}{3\pi}\right) + (40 \cdot 30) \cdot 15}{\frac{\pi 40^2}{4 \cdot 2} + 40 \cdot 30} = 23{,}07$$

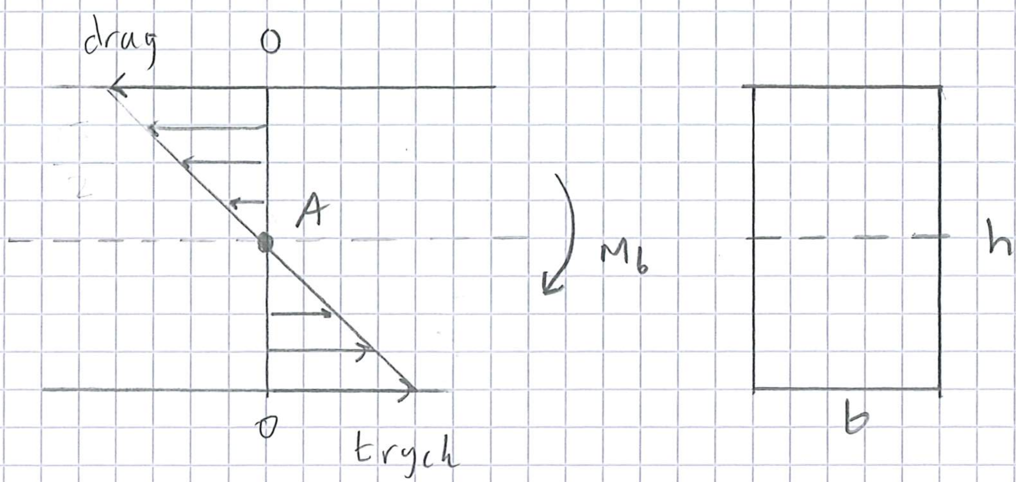
$$I_y = I_1 + A_1 a_1^2 + I_2 + A_2 a_1^2 =$$

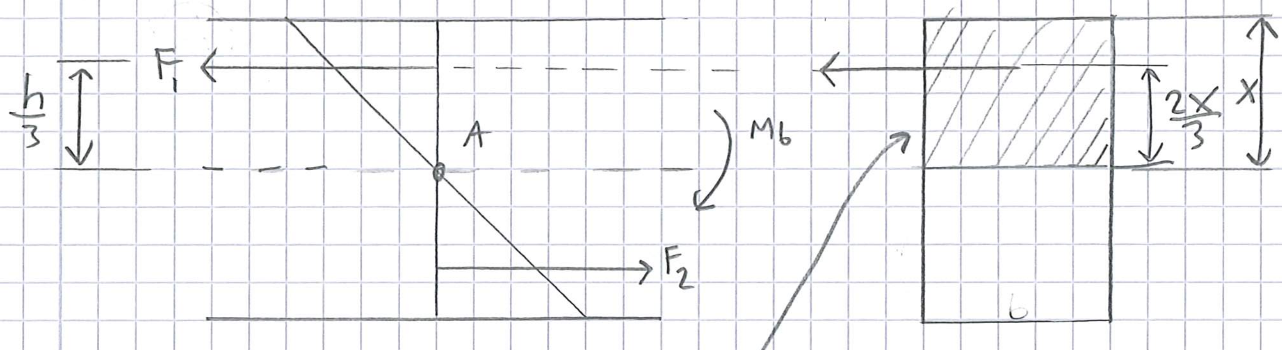
$$= \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) 20^4 + \frac{\pi 40^2}{8} \cdot \left(23 - \left(30 + \frac{4 \cdot 20}{3\pi}\right)\right)^2 + \frac{40 \cdot 30^3}{12} + (40 \cdot 30) \cdot (23 - 15)^2 =$$

$$= \underline{\underline{335086}} \text{ mm}^4$$

$$W_{by} = \frac{I_y}{e_{max}} = \frac{335\,086}{50 - 23{,}07} = \underline{\underline{12{,}4 \text{ cm}^3}}$$

17





$x = \frac{h}{2}$

$\frac{2x}{3} = \frac{2}{3}\cdot\frac{h}{2} = \frac{h}{3}$

Kraften F verkar på arean $b\cdot\frac{h}{2}$

$$F_1 = F_2 = F = \sigma\cdot A = \frac{\sigma_b}{2}\cdot\frac{b\,h}{2}$$

Halva böjspänningen finns på ovansidan och den andra halvan på undersidan. ($\sigma_b/2$ verkar på vardera sida om neutrallinjen)

$$\curvearrowleft A: \; F_1\frac{h}{3} + F_2\cdot\frac{h}{3} - M_b = 0$$

$$2\cdot F\frac{h}{3} - M_b = 0$$

$$2\left(\frac{\sigma_b}{2}\cdot\frac{b\cdot h}{2}\right)\cdot\frac{h}{3} = M_b$$

$$\sigma_b\cdot\frac{b\cdot h^2}{6} = M_b$$

$$\sigma_b = \frac{M_b}{\frac{b\cdot h^2}{6}} = \frac{M_b}{W_b}$$ där $W_b = \frac{b\cdot h^2}{6}$