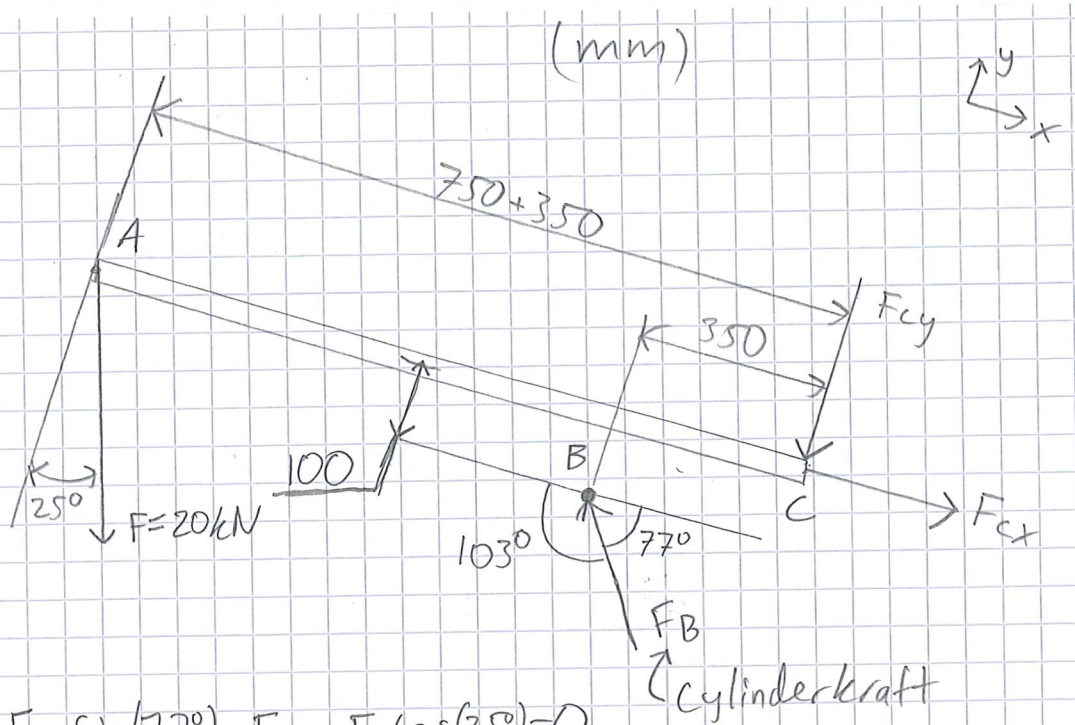


(21)



$$\begin{aligned}
 \text{I} & \left\{ \begin{aligned} \uparrow: F_B \cdot \sin(77^\circ) - F_{cy} - F \cdot \cos(25^\circ) &= 0 \\ \rightarrow: F_{cx} + F \cdot \sin(25^\circ) - F_B \cos(77^\circ) &= 0 \\ \curvearrowright: F_B \cdot \sin(77^\circ) \cdot 350 + F_B \cdot \cos(77^\circ) \cdot 100 - F \cdot \cos(25^\circ) \cdot (750 + 350) &= 0 \end{aligned} \right.
 \end{aligned}$$

• ekv III ger  $F_B$ :

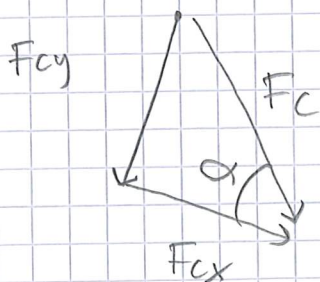
$$F_B = \frac{F \cdot \cos(25^\circ) \cdot (750 + 350)}{\sin(77^\circ) \cdot 350 + \cos(77^\circ) \cdot 100} = \frac{20\,000 \cdot \cos(25^\circ) \cdot (750 + 350)}{\sin(77^\circ) \cdot 350 + \cos(77^\circ) \cdot 100} \approx 54\,848,5 \text{ N}$$

• ekv II ger  $F_{cx}$ :

$$F_{cx} = F_B \cdot \cos(77^\circ) - F \cdot \sin(25^\circ) = 54\,848,5 \cdot \cos(77^\circ) - 20\,000 \cdot \sin(25^\circ) \approx 3\,885,9 \text{ N}$$

• ekv I ger  $F_{cy}$ :

$$F_{cy} = F_B \sin(77^\circ) - F \cdot \cos(25^\circ) = 54\,848,5 \cdot \sin(77^\circ) - 20\,000 \cdot \cos(25^\circ) \approx 35\,316,6 \text{ N}$$



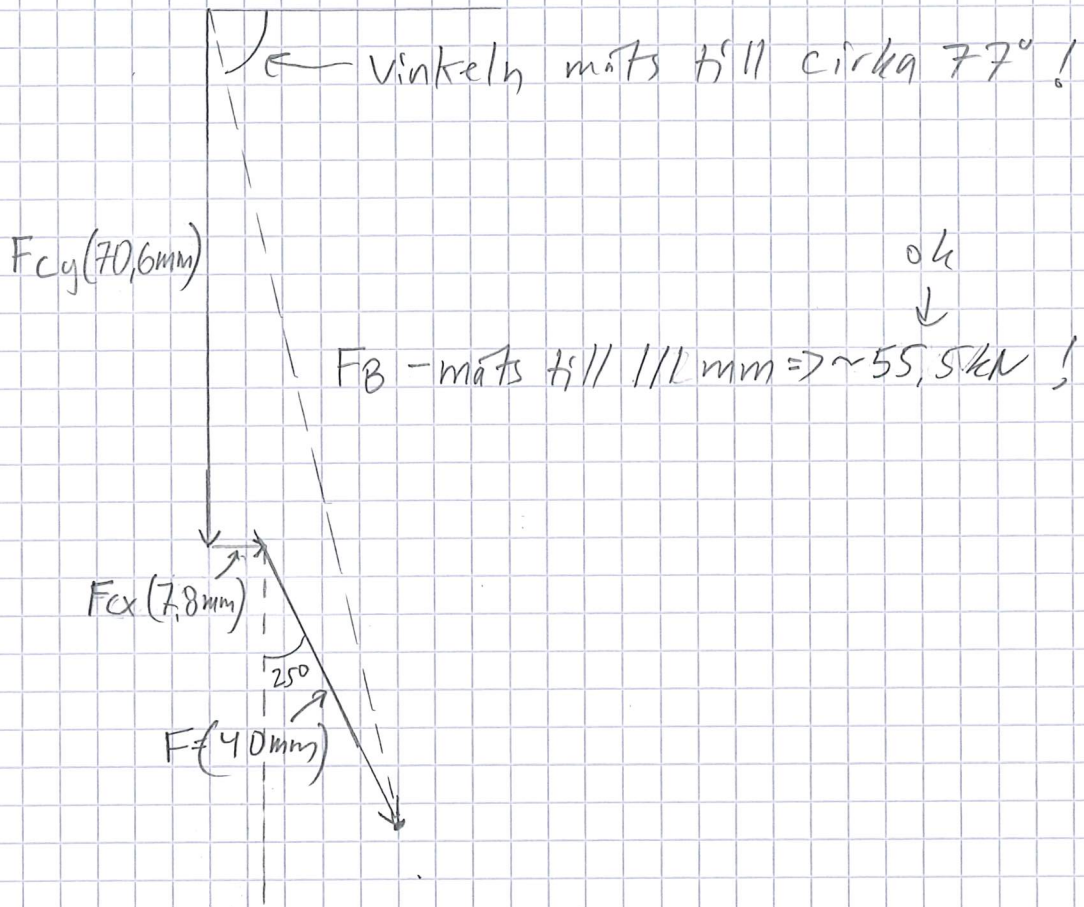
$$F_c = \sqrt{F_{cy}^2 + F_{cx}^2} = \sqrt{35\,316,6^2 + 3\,885,9^2} \approx 35\,529,7 \text{ N}$$

$$\alpha = \arctan\left(\frac{F_{cy}}{F_{cx}}\right) = \arctan\left(\frac{35\,316,6}{3\,885,9}\right) \approx 83,7^\circ$$

$\boxed{1/2}$

21

Skala: 1 kN = 2 mm

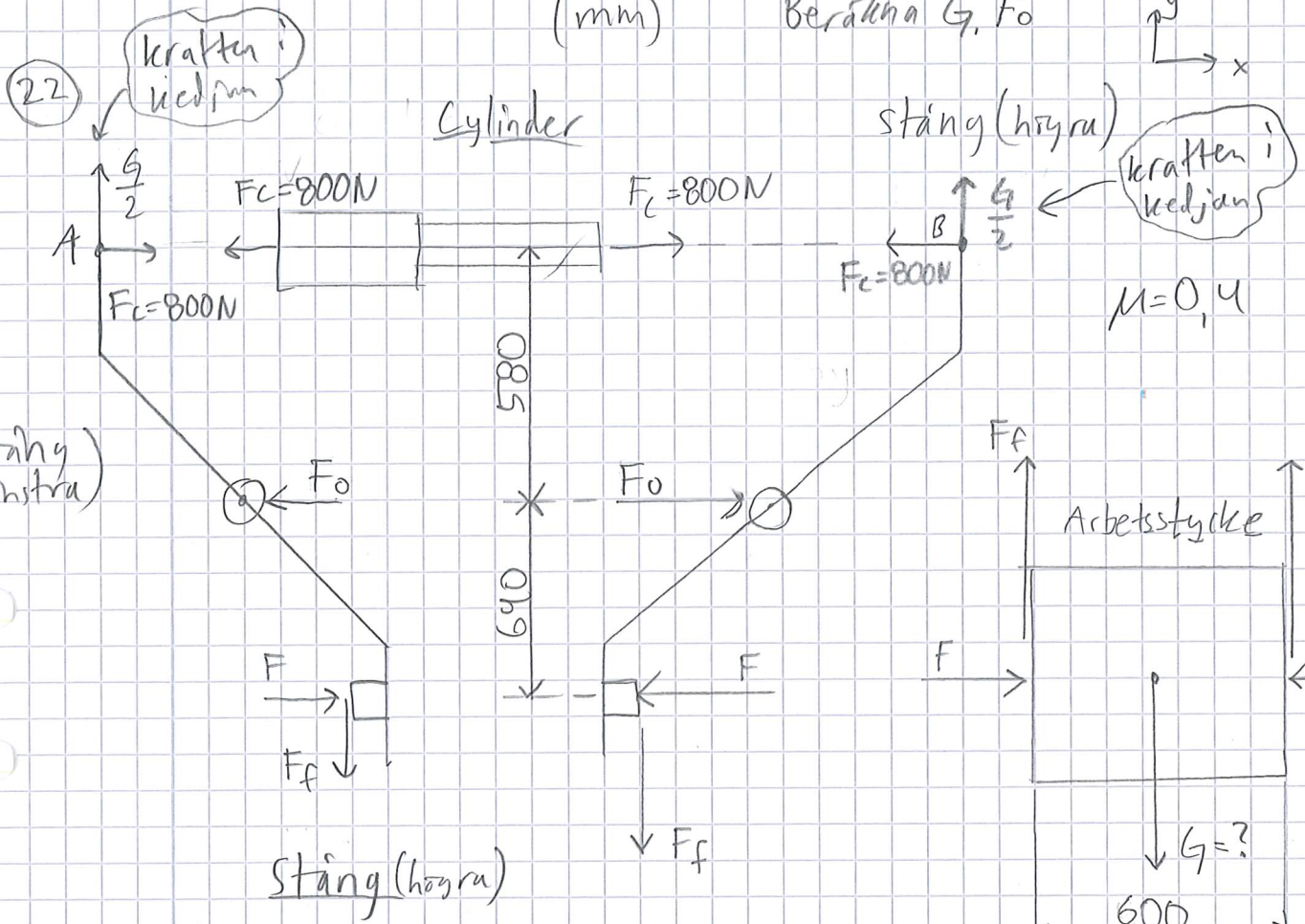
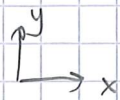


Svar: Cylinderkraften är cirka 55 kN och reaktionskraften vid C cirka 35,5 kN, med cirka vinkeln  $84^\circ$ .

2/2

(mm)

Beräkna  $G, F_0$



I  $\left\{ \begin{array}{l} \uparrow: G/2 - F_f = 0 \end{array} \right.$

II  $\left\{ \begin{array}{l} \rightarrow: F_0 - F_c - F = 0 \end{array} \right.$

III  $\left\{ \begin{array}{l} \circlearrowleft: F \cdot 640 - F_c \cdot 580 - F_f \cdot 300 = 0 \end{array} \right.$

IV  $\left\{ \begin{array}{l} F_f = \mu \cdot F \end{array} \right.$

V  $\left\{ \begin{array}{l} \uparrow: 2F_f - G = 0 \end{array} \right.$

VI  $\left\{ \begin{array}{l} \rightarrow: F - F = 0 \end{array} \right.$

• ekvation III och IV ger F

$$F \cdot 640 - F_c \cdot 580 - \underbrace{\mu \cdot F \cdot 300}_{F_f} = 0$$

$$F = \frac{F_c \cdot 580}{640 - \mu \cdot 300} = \frac{800 \cdot 580}{640 - 0,4 \cdot 300} \approx 892,3 \text{ N}$$

• ekvation V ger  $G$ :

$$G = 2 \cdot F_f = 2 \cdot \mu \cdot F = 2 \cdot 0,4 \cdot 892,3 \approx 713,8 \text{ N}$$

1/2

22

• ekvation II ger  $F_0$ :

$$F_0 = F + F_c = 892,3 + 800 = 1692,3 \text{ N}$$

• ekvation I kan användas för kontroll:

$$F_f = \mu \cdot F = 0,4 \cdot 892,3 \approx 356,92 \text{ N}$$

$$I \quad \frac{G}{2} - F_f = 0$$

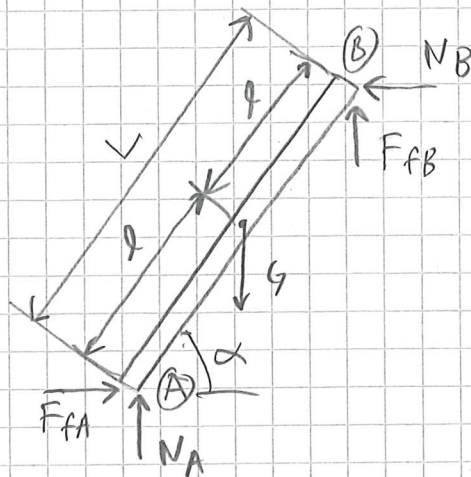
$$\frac{713,8}{2} - 356,92 \approx 0 \quad \text{ok!}$$

Svar: kraften  $G$  kan vara max 710 N och reaktionskraften vid 0 är cirka 1,74 N

23

$$L = 2l$$

$$\mu = 0,3$$



$$\textcircled{I} \uparrow: N_A - G + F_{fB} = 0 \Rightarrow N_A - G + N_B \cdot \mu = 0$$

$$\textcircled{II} \rightarrow: F_{fA} - N_B = 0 \quad N_A \cdot \mu - N_B = 0$$

$$\overset{\curvearrowright}{M}_A: G \cdot l \cdot \cos(\alpha) - F_{fB} \cdot 2l \cos(\alpha) - N_B \cdot 2l \sin(\alpha) = 0$$

$$G \cdot l \cdot \cos(\alpha) - N_B \cdot \mu \cdot 2l \cos(\alpha) - N_B \cdot 2l \sin(\alpha) = 0$$

delar med  $l \cos(\alpha)$

$$G - N_B \mu \cdot 2 - N_B \cdot 2 \tan(\alpha) = 0$$

$$G - 2\mu N_B - 2N_B \tan(\alpha) = 0$$

$$\textcircled{II} N_A = \frac{N_B}{\mu}$$

$$\textcircled{I} N_A - G + N_B \cdot \mu = 0$$

$$N_B = \frac{G - N_A}{\mu} = \frac{G - \frac{N_B}{\mu}}{\mu}$$

$$\mu \cdot N_B + \frac{N_B}{\mu} = G \Rightarrow N_B = \frac{G}{\mu + \frac{1}{\mu}}$$

$\boxed{1/2}$

23

forts...

$$G - 2\mu \left[ \frac{G}{M+L} \right] - 2 \left[ \frac{G}{M+L} \right] \cdot \tan(\alpha) = 0$$

dela med G

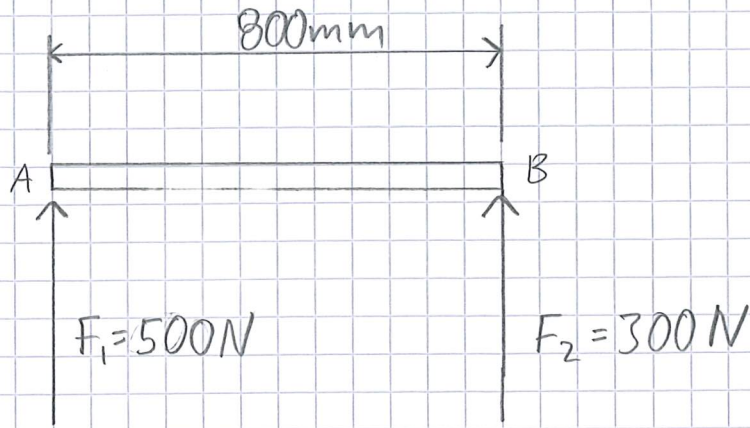
$$1 - \frac{2\mu}{M+L} - \frac{2}{M+L} \cdot \tan(\alpha) = 0$$

$$\alpha = \arctan \left( \frac{1 + \frac{2 \cdot 0,3}{0,3 + 1}}{\frac{2}{0,3 + 1}} \right) = \underline{\underline{56,6^\circ}}$$

2/2

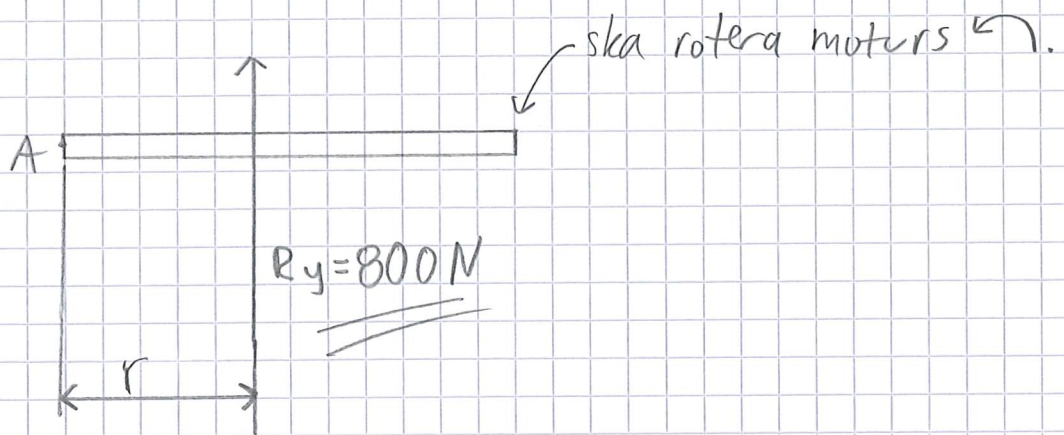
24)

d)



$$\uparrow R_y: F_1 + F_2 = 500 + 300 = 800\text{ N}$$

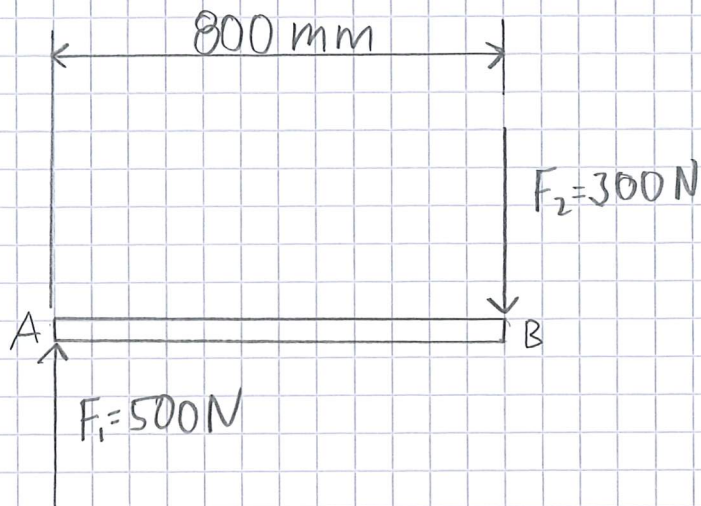
$$(M_A) \curvearrowleft A: F_2 \cdot 800 = 300 \cdot 800 = 240\,000\text{ Nmm}$$



$$M_A = R_y \cdot r \Rightarrow r = \frac{M_A}{R_y} = \frac{240\,000}{800} = \underline{\underline{300\text{ mm}}}$$

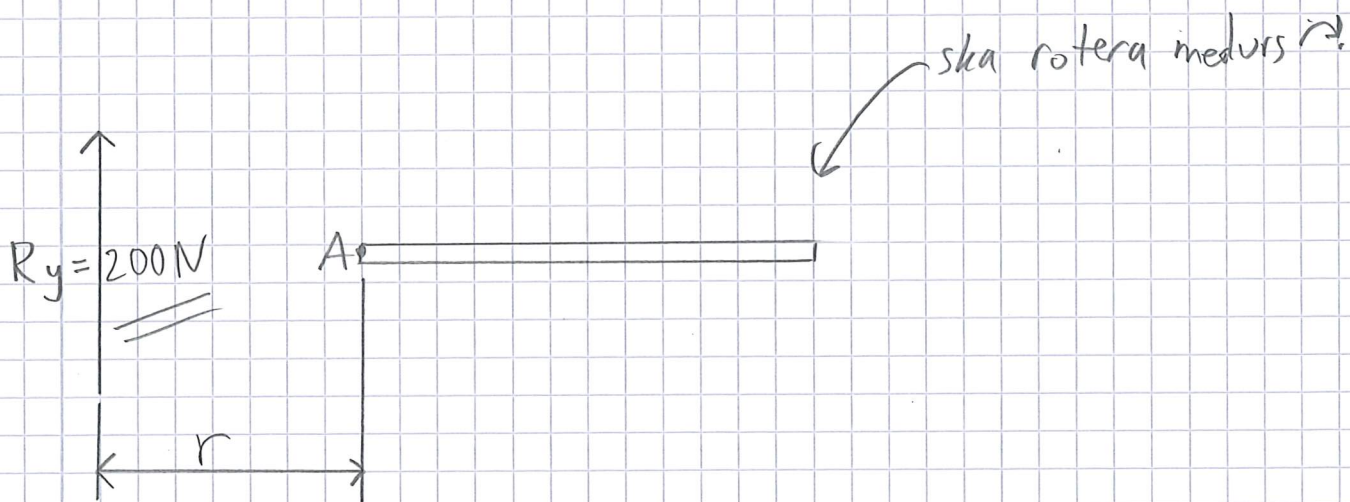
(24)

b)



$$\uparrow R_y: F_1 - F_2 = 500 - 300 = 200\text{ N}$$

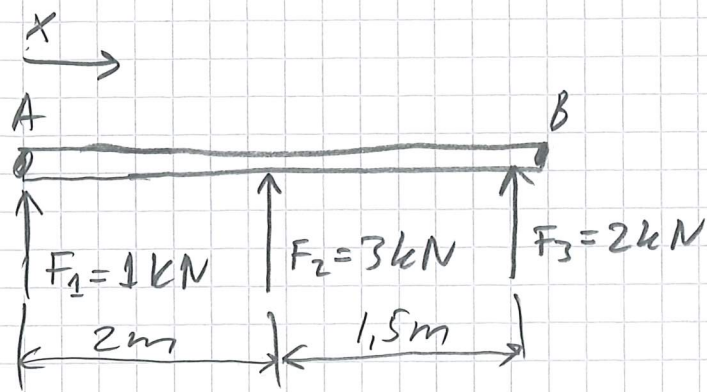
$$(M_A) \curvearrowright A: F_2 \cdot 800 = 300 \cdot 800 = 240\,000\text{ Nmm}$$



$$M_A = R_y \cdot r \Rightarrow r = \frac{M_A}{R_y} = \frac{240\,000}{200} = \underline{\underline{1200\text{ mm}}}$$



(25)

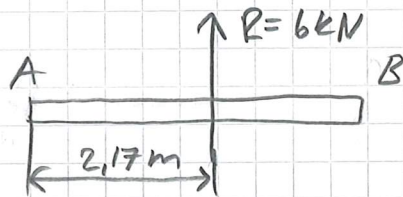


Bestäm resultanten a)  $F_1, F_2, F_3$  går åt samma håll  
 b)  $F_2$  går åt motsatt håll  $F_1$  &  $F_3$

a)  $\uparrow R_y: F_1 + F_2 + F_3 = 1 + 3 + 2 = \underline{6 \text{ kN}}$

$\curvearrowleft M_A: F_2 \cdot 2 + F_3 \cdot (2 + 1.5) = 3 \cdot 2 + 2 \cdot 3.5 = 13 \text{ kNm}$

$M_A = r \cdot R_y \quad r = \frac{M_A}{R_y} = \frac{13}{6} = \underline{2.17 \text{ m}}$

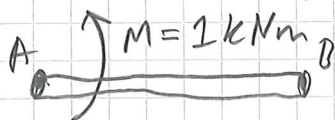


b) om  $F_2$  byter riktning.

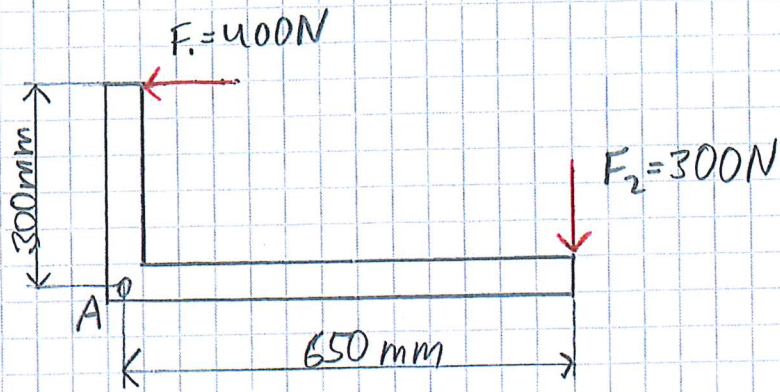
$\uparrow R_y: F_1 - F_2 + F_3 = 1 - 3 + 2 = \underline{0 \text{ kN}}$

$\curvearrowleft M_A: -F_2 \cdot 2 + F_3 \cdot (2 + 1.5) = -3 \cdot 2 + 2 \cdot 3.5 = \underline{1 \text{ kNm}}$

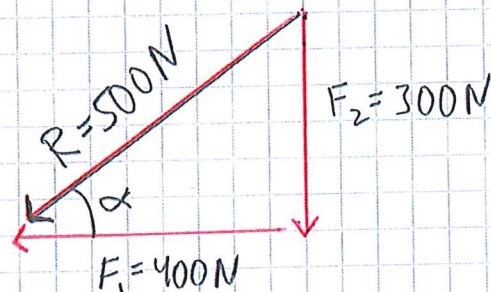
$M_A = r \cdot R_y \Rightarrow r = \frac{M_A}{R_y} = \frac{1}{0} = \text{ej def.}$



26



Kraftresultanten blir:

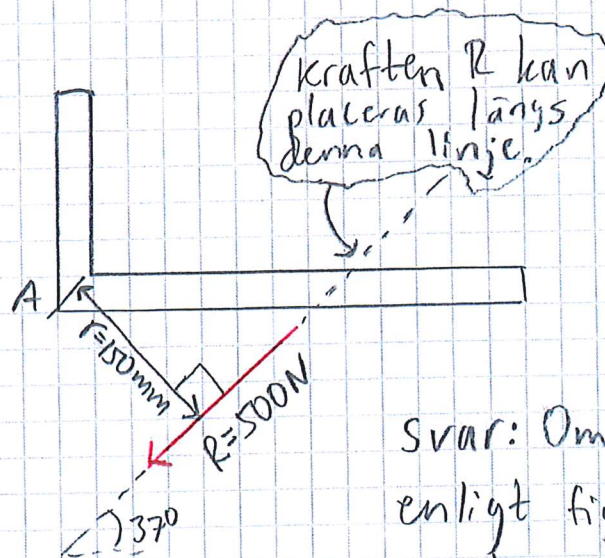


$$R = \sqrt{300^2 + 400^2}$$

$$\alpha = \arctan\left(\frac{300}{400}\right) \approx 36,9^\circ$$

Momentresultanten blir:

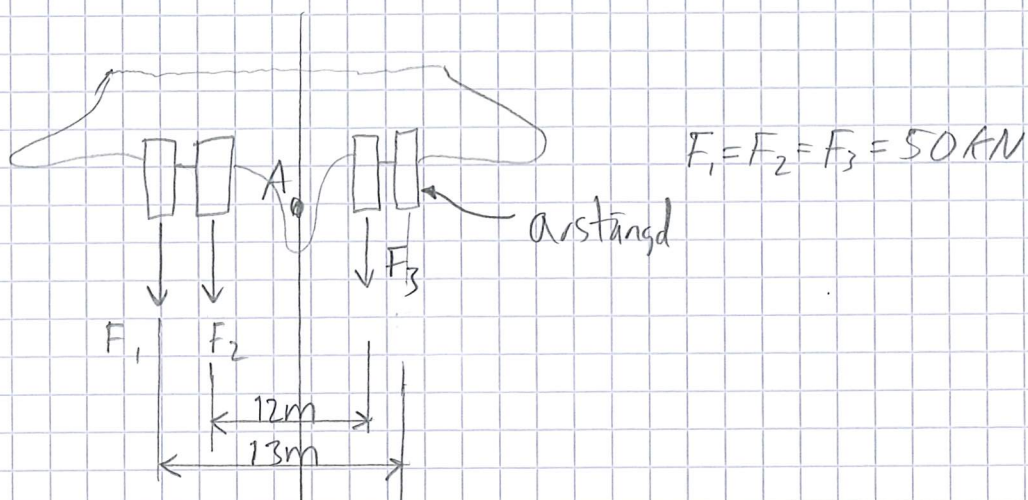
$$A^{\curvearrowright} = 300 \cdot 0,650 - 400 \cdot 0,300 = 75,0 \text{ Nm}$$



$$r = \frac{A^{\curvearrowright}}{R} = \frac{75,0}{500} = 150\text{ mm}$$

Svar: Om kraften placeras enligt figuren ovan blir kraft- och momentresultanterna densamma som i översta figuren.

27

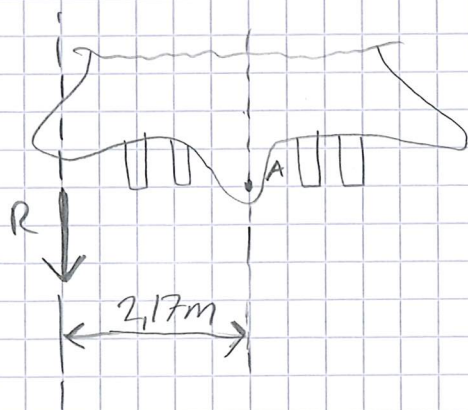


$$(\downarrow) R = F_1 + F_2 + F_3 = 50 + 50 + 50 = 150 \text{ kN}$$

$$(\curvearrowright) M = F_2 \cdot 6 + F_1 \cdot 6,5 - F_3 \cdot 6 =$$

$$= 50 \cdot 6 + 50 \cdot 6,5 - 50 \cdot 6 = 325 \text{ kNm}$$

$$M = r \cdot R \Rightarrow r = \frac{M}{R} = \frac{325}{150} \approx 2,17 \text{ m}$$



Öka  $F_3$  så att  $M = 0$

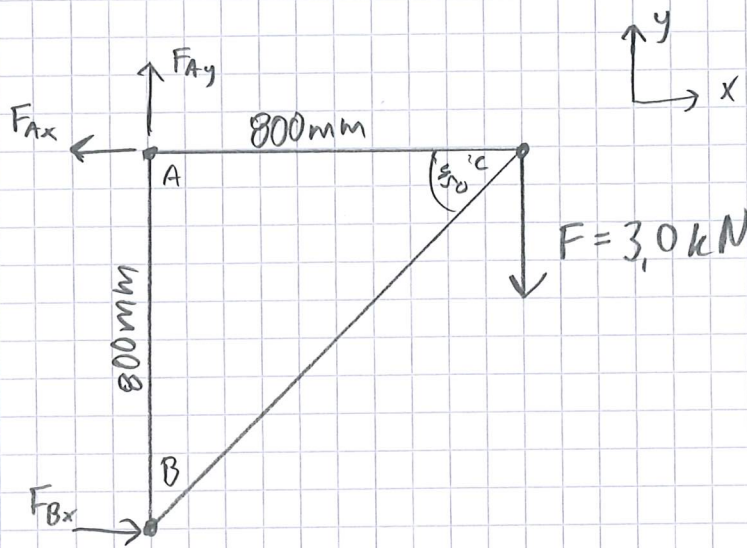
här ska momentet  
vara lika med 0

$$(\curvearrowright) M = F_2 \cdot 6 + F_1 \cdot 6,5 - F_3 \cdot 6 = 0$$

$$F_3 = \frac{F_2 \cdot 6 + F_1 \cdot 6,5}{6} = \frac{50 \cdot 6 + 50 \cdot 6,5}{6} \approx 104,17 \text{ kN}$$

(28)

### Hela systemet

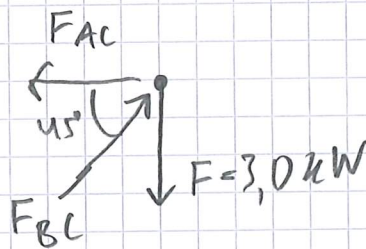


$$\uparrow: F_{Ay} - F = 0 \Rightarrow F_{Ay} = F = 3,0 \text{ kN}$$

$$\rightarrow: F_{Bx} - F_{Ax} = 0 \Rightarrow F_{Bx} = F_{Ax} = 3,0 \text{ kN}$$

$$\curvearrow: F \cdot 800 - F_{Bx} \cdot 800 = 0 \Rightarrow F_{Bx} = F = 3,0 \text{ kN} \text{ och}$$

### Punkt C



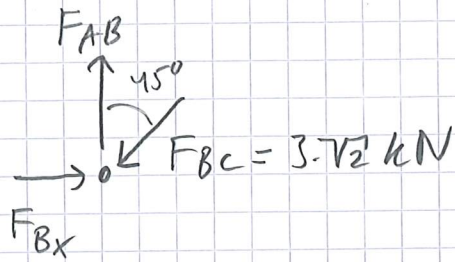
$$\left\{ \begin{array}{l} \uparrow: F_{BC} \cdot \sin(45^\circ) - F = 0 \\ \rightarrow: F_{BC} \cdot \cos(45^\circ) - F_{AC} = 0 \end{array} \right. \Rightarrow F_{BC} = \frac{F}{\sin(45^\circ)} = \frac{3,0}{\frac{1}{\sqrt{2}}} = 3 \cdot \sqrt{2} \text{ kN}$$

$$\Rightarrow F_{AC} = F_{BC} \cdot \cos(45^\circ) = 3 \cdot \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 3 \text{ kN}$$

1/2

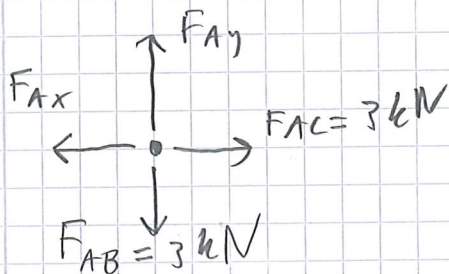
(28)

Punkt B



$$\begin{cases} \uparrow: F_{AB} - F_{BC} \cdot \cos(45^\circ) = 0 \Rightarrow F_{AB} = F_{BC} \cdot \cos(45^\circ) = 3 \cdot \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 3 \text{ kN} \\ \rightarrow: F_{Bx} - F_{BC} \cdot \sin(45^\circ) = 0 \Rightarrow F_{Bx} = F_{BC} \cdot \sin(45^\circ) = 3 \cdot \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 3 \text{ kN} \end{cases}$$

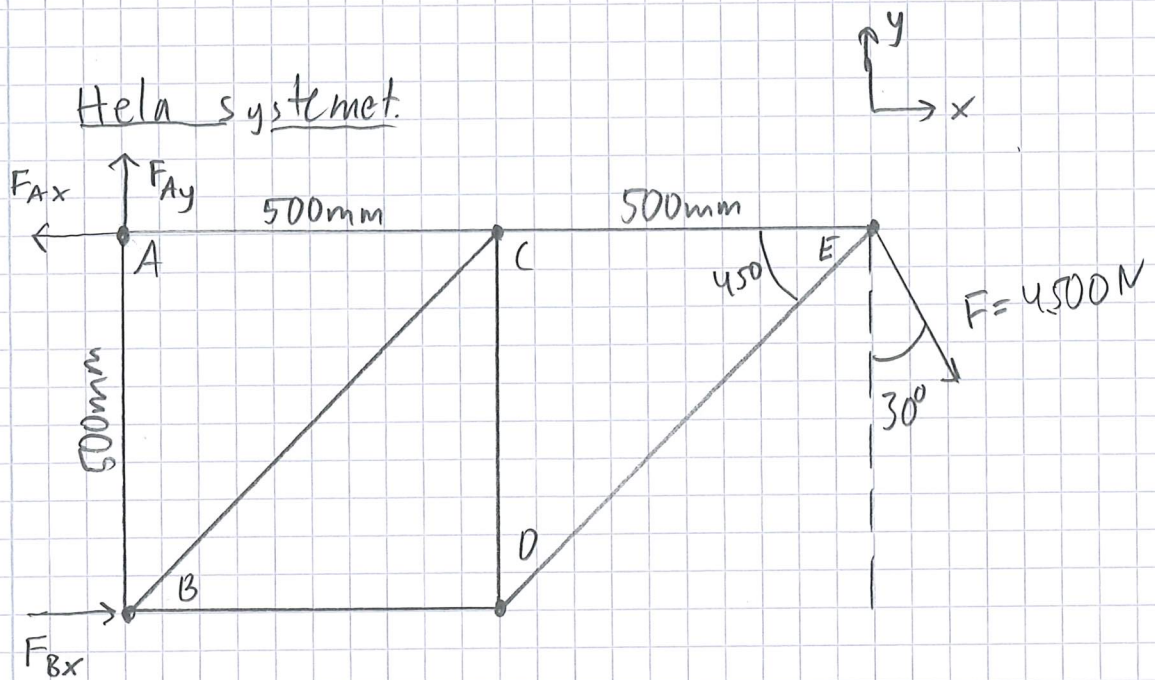
Punkt C



$$\begin{aligned} \uparrow: F_{Ay} - F_{AB} &= 0 \Rightarrow F_{Ay} = F_{AB} = 3 \text{ kN} \\ \rightarrow: F_{AC} - F_{Ax} &= 0 \Rightarrow F_{AC} = F_{Ax} = 3 \text{ kN} \end{aligned}$$

(29)

Hela systemet.



$$\begin{array}{l} \text{I} \\ \text{II} \\ \text{III} \end{array} \left\{ \begin{array}{l} \uparrow: F_{Ay} - F \cdot \cos(30^\circ) = 0 \\ \rightarrow: F_{Bx} + F \cdot \sin(30^\circ) - F_{Ax} = 0 \\ \curvearrowright: F \cdot \cos(30^\circ) \cdot 1000 - F_{Bx} \cdot 500 = 0 \end{array} \right.$$

ekv I ger:  $F_{Ay} = F \cdot \cos(30^\circ) = 4500 \cdot \cos(30^\circ) \approx 3897,1 \text{ N}$

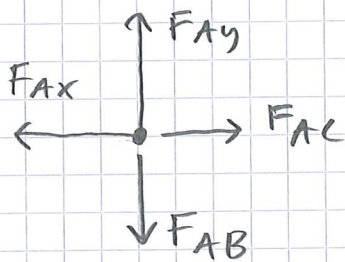
ekv III ger:  $F_{Bx} = \frac{F \cdot \cos(30^\circ) \cdot 1000}{500} = \frac{4500 \cdot \cos(30^\circ) \cdot 1000}{500} \approx 7794,2 \text{ N}$

ekv II ger nu:  $F_{Ax} = F_{Bx} + F \sin(30^\circ) =$

$$= 7794,2 + 4500 \cdot \sin(30^\circ) \approx 10044 \text{ N}$$

(29)

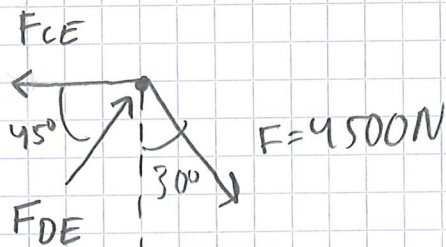
## Punkt A



$$\uparrow: F_{Ay} - F_{AB} = 0 \Rightarrow F_{Ay} = F_{AB} = 3897,1 \text{ N}$$

$$\rightarrow: F_{AC} - F_{Ax} = 0 \Rightarrow F_{AC} = F_{Ax} = 10\,044 \text{ N}$$

Punkt E (börja med denna eftersom kratternas riktningar är kända här)



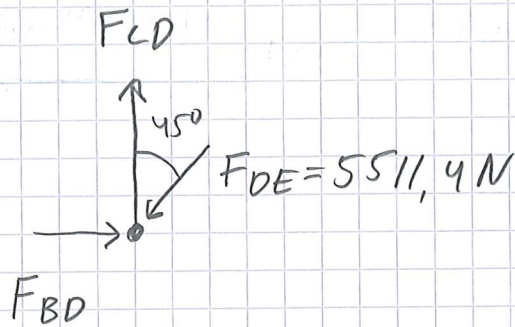
$$\text{I} \left\{ \begin{array}{l} \uparrow: F_{DE} \cdot \sin(45^\circ) - F \cdot \cos(30^\circ) = 0 \end{array} \right.$$

$$\text{II} \left\{ \begin{array}{l} \rightarrow: F_{DE} \cos(45^\circ) + F \cdot \sin(30^\circ) - F_{CE} = 0 \end{array} \right.$$

$$\text{ekv I ger: } F_{DE} = F \cdot \frac{\cos(30^\circ)}{\sin(45^\circ)} = \frac{4500 \cdot \cos(30^\circ)}{\sin(45^\circ)} \approx 5511,4 \text{ N}$$

$$\begin{aligned} \text{ekv II ger: } F_{CE} &= F_{DE} \cdot \cos(45^\circ) + F \cdot \sin(30^\circ) = \\ &= 5511,4 \cdot \cos(45^\circ) + 4500 \cdot \sin(30^\circ) \approx 6147,1 \text{ N} \end{aligned}$$

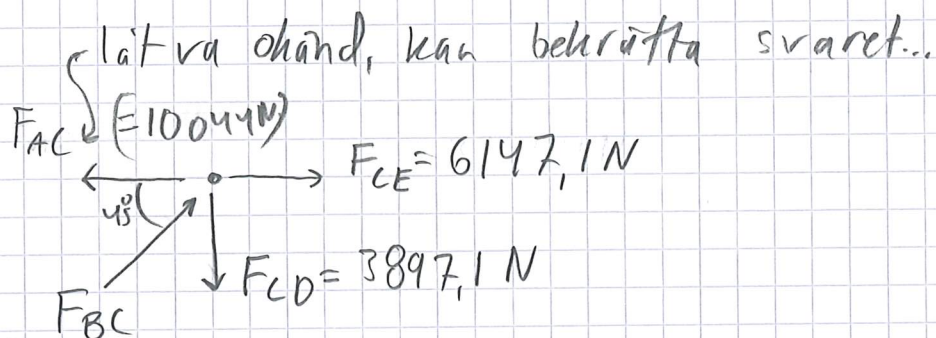
(29)

Punkt D

$$\begin{cases} \text{I} \left\{ \begin{array}{l} \uparrow: F_{CD} - F_{DE} \cdot \cos(45^\circ) = 0 \\ \text{II} \left\{ \begin{array}{l} \rightarrow: F_{BD} - F_{DE} \cdot \sin(45^\circ) = 0 \end{array} \right. \end{array} \right. \end{cases}$$

ekv I ger:  $F_{CD} = F_{DE} \cdot \cos(45^\circ) = 5511,4 \cdot \cos(45^\circ) \approx 3897,1 \text{ N}$

ekv II ger:  $F_{BD} = F_{DE} \cdot \sin(45^\circ) = 5511,4 \cdot \sin(45^\circ) \approx 3897,1 \text{ N}$

Punkt C

$$\begin{cases} \text{I} \left\{ \begin{array}{l} \uparrow: F_{BC} \cdot \sin(45^\circ) - F_{CD} = 0 \\ \text{II} \left\{ \begin{array}{l} \rightarrow: F_{BC} \cdot \cos(45^\circ) + F_{CE} - F_{AC} = 0 \end{array} \right. \end{array} \right. \end{cases}$$

ekv I:  $F_{BC} = \frac{F_{CD}}{\sin(45^\circ)} = \frac{3897,1}{\sin(45^\circ)} \approx 5511,3 \text{ N}$

3/4
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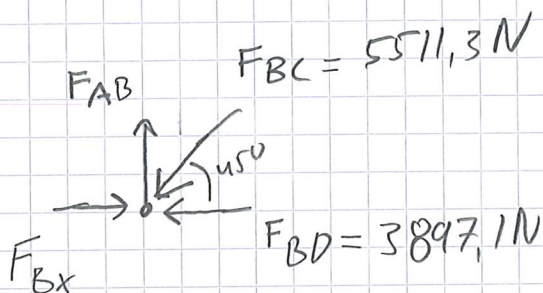


29

$$\begin{aligned} \text{ekv II ger: } F_{Ax} &= F_{Bc} \cdot \cos(45^\circ) + F_{CE} \\ &= 5511,3 \cdot \cos(45^\circ) + 6147,1 \text{ N} \approx 10044 \text{ N} \end{aligned}$$

Punkt B

Ok, stämmer bra!



Låt  $F_{Bx}$  och  $F_{AB}$  vara obekanta för kontroll.

$$\begin{cases} \text{I} & \uparrow: F_{AB} - F_{Bc} \cdot \sin(45^\circ) = 0 \\ \text{II} & \rightarrow: F_{Bx} - F_{BD} - F_{Bc} \cdot \cos(45^\circ) = 0 \end{cases}$$

$$\text{ekv I ger: } F_{AB} = F_{Bc} \cdot \sin(45^\circ) = 5511,3 \cdot \sin(45^\circ) \approx 3897,1 \text{ N}$$

Ok, stämmer bra!

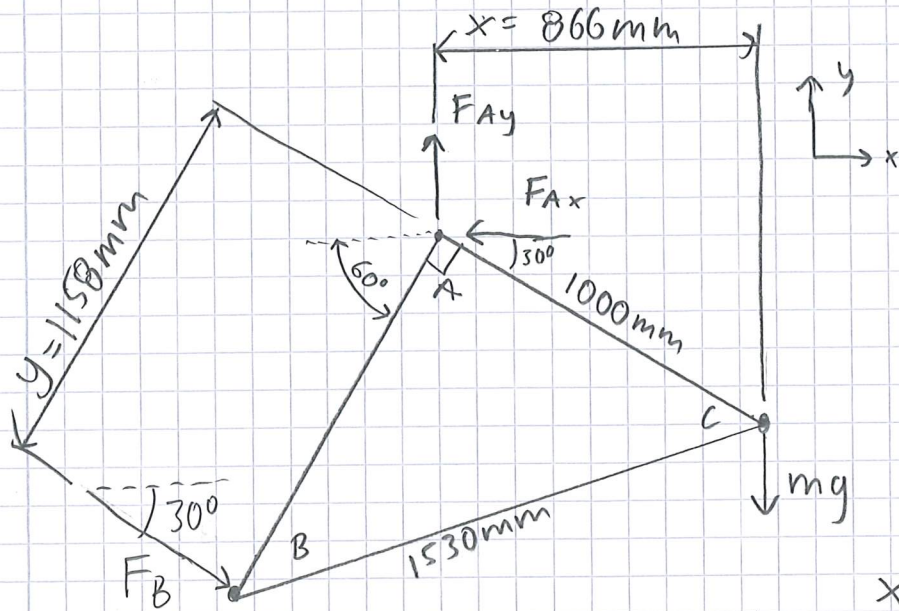
$$\text{ekv II ger: } F_{Bx} = F_{Bc} \cdot \cos(45^\circ) + F_{BD} = 5511,3 \cdot \cos(45^\circ) + 3897,1 \approx$$

$$\approx 7794,2 \text{ N}$$

Ok, stämmer bra!

4/4

30



$$m = 500 \text{ kg}$$

$$x = 1000 \cdot \cos(30^\circ) \approx 866 \text{ mm}$$

$$y = \sqrt{1530^2 - 1000^2} \approx 1158 \text{ mm}$$

$$\begin{cases} \text{I} & \uparrow: F_{Ay} - F_B \cdot \sin(30^\circ) - m \cdot g = 0 \\ \text{II} & \rightarrow: F_B \cdot \cos(30^\circ) - F_{Ax} = 0 \\ \text{III} & \curvearrowright: m \cdot g \cdot 866 - F_B \cdot 1158 = 0 \end{cases}$$

$$\text{ekv III ger: } F_B = \frac{m \cdot g \cdot 866}{1158} = \frac{500 \cdot 10 \cdot 866}{1158} \approx 3739,2 \text{ N}$$

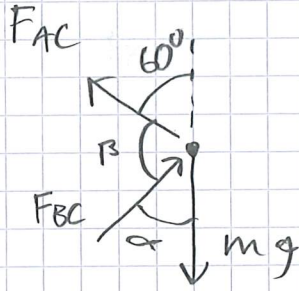
$$\text{ekv II ger: } F_{Ax} = F_B \cdot \cos(30^\circ) = 3739,2 \cdot \cos(30^\circ) \approx 3238,2 \text{ N}$$

$$\text{ekv I ger: } F_{Ay} = m \cdot g + F_B \sin(30^\circ) = 500 \cdot 10 + 3739,2 \cdot \sin(30^\circ) \approx 6869,6 \text{ N}$$

1/4

30

### punkt C



$$\beta = \arctan\left(\frac{1158}{1000}\right) \approx 49,2^\circ$$

$$\alpha = 180 - \beta - 60 = 70,8^\circ$$

$$\begin{cases} \text{I} \quad \uparrow: F_{AC} \cdot \cos(60^\circ) - mg + F_{BC} \cdot \cos(70,8^\circ) = 0 \\ \text{II} \quad \rightarrow: F_{BC} \cdot \sin(70,8^\circ) - F_{AC} \cdot \sin(60^\circ) = 0 \end{cases}$$

ekv II ger:  $F_{BC} = F_{AC} \cdot \frac{\sin(60^\circ)}{\sin(70,8^\circ)}$

ekv II; I ger

$$F_{AC} \cos(60^\circ) - mg + \overbrace{F_{AC} \frac{\sin(60^\circ)}{\sin(70,8^\circ)}}^{F_{BC}} \cdot \cos(70,8^\circ) = 0$$

$$F_{AC} = \frac{mg}{\cos(60^\circ) + \frac{\sin(60^\circ)}{\sin(70,8^\circ)} \cdot \cos(70,8^\circ)} \approx 6237,7 \text{ N}$$

Om  $mg = 5000 \text{ N}$

Om  $F_{AC} = 6238 \text{ N}$

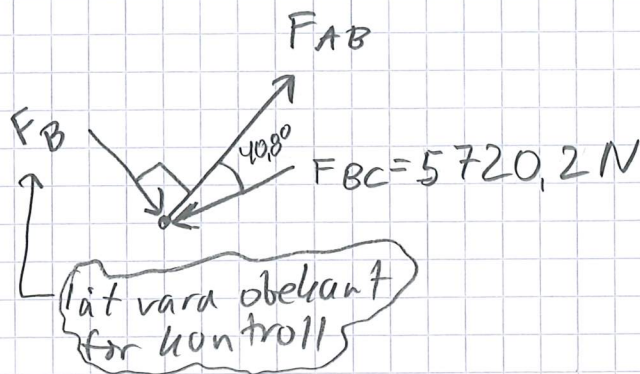
$$\Rightarrow F_{BC} = F_{AC} \cdot \frac{\sin(60^\circ)}{\sin(70,8^\circ)} \approx 5720,2 \text{ N}$$

30

Punkt B



$$180 - 90 - 49,2 = 40,8^\circ$$



$$\begin{cases} \text{I} & \rightarrow: F_{AB} - F_{BC} \cdot \cos(40,8^\circ) = 0 \\ \text{II} & \downarrow: F_B - F_{BC} \cdot \sin(40,8^\circ) = 0 \end{cases}$$

$$\text{ekv I ger } F_{AB} = F_{BC} \cdot \cos(40,8^\circ) = 5720,2 \cdot \cos(40,8^\circ) \approx \\ \approx 4330,2 \text{ N}$$

$$\text{ekv II ger } F_B = F_{BC} \cdot \sin(40,8^\circ) = 5720,2 \cdot \sin(40,8^\circ) \approx \\ \approx 3737,7 \text{ N}$$

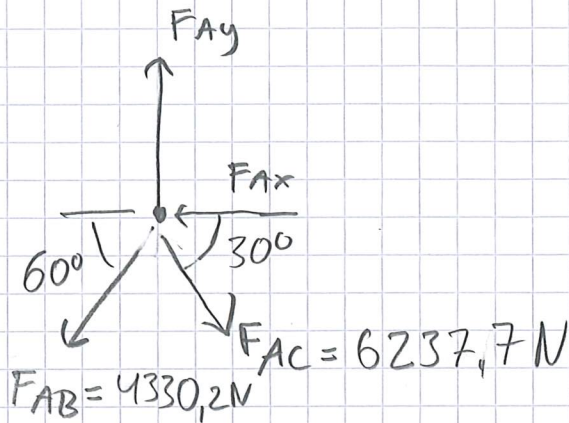
↑  
ok, stämmer (med  
visst avrundningsfel)

Testa gärna att räkna knutpunkterna ut  
att avrunda, då stämmer svaret till 100%!

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## Punkt A



$$\text{I} \quad \left\{ \begin{array}{l} \uparrow: F_{Ay} - F_{AB} \sin(60^\circ) - F_{AC} \cdot \sin(30^\circ) = 0 \end{array} \right.$$

$$\text{II} \quad \left\{ \begin{array}{l} \rightarrow: F_{AC} \cdot \cos(30^\circ) - F_{AB} \cdot \cos(60^\circ) - F_{Ax} = 0 \end{array} \right.$$

ekv II ger:  $F_{Ax} = F_{AC} \cdot \cos(30^\circ) - F_{AB} \cdot \cos(60^\circ) =$

$$= 6237,7 \cdot \cos(30^\circ) - 4330,2 \cdot \cos(60^\circ) \approx 3236,9 \text{ N}$$

~ok!  
↓

ekv I ger:  $F_{Ay} = F_{AC} \cdot \sin(30^\circ) + F_{AB} \sin(60^\circ) =$

$$= 6238 \cdot \sin(30^\circ) + 4330,2 \cdot \sin(60^\circ) \approx 6869,1 \text{ N}$$

↑  
~ok!

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