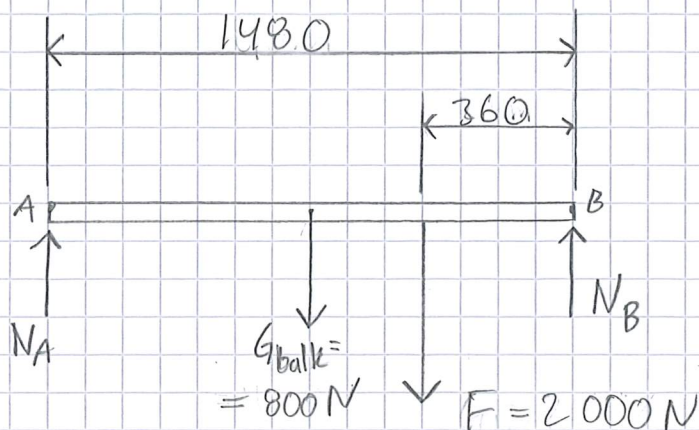


4)



$$\begin{cases} \text{I} & \uparrow: N_A + N_B - G_{\text{balk}} - F = 0 \\ \text{II} & \curvearrow B: N_A \cdot 1480 - G_{\text{balk}} \cdot \frac{1480}{2} - F \cdot 360 = 0 \end{cases}$$

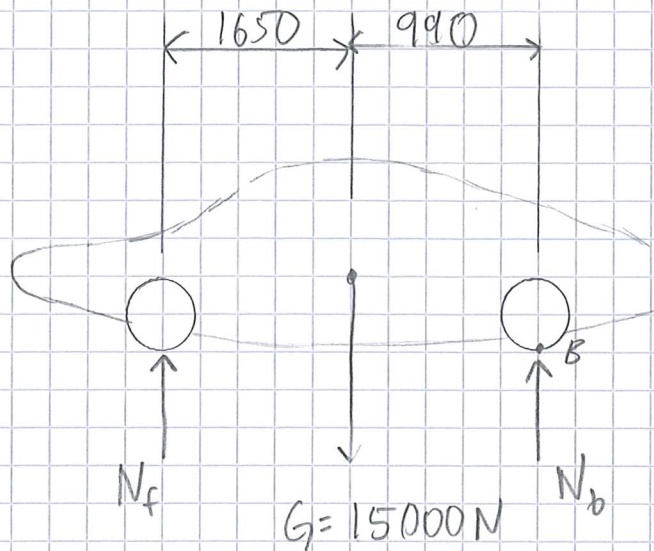
$$\text{ekv II ger: } N_A = \frac{F \cdot 360 + G_{\text{balk}} \cdot \frac{1480}{2}}{1480} \approx 886,5 \text{ N}$$

$$\text{ekv I ger: } N_B = G_{\text{balk}} + F - N_A = 800 + 2000 - 886,5 = 1913,5 \text{ N}$$

Svar: N_A är cirka 887 N och N_B 1914 N.

5

(mm)



$$\text{I: } \left\{ \begin{array}{l} \uparrow: N_f + N_b - G = 0 \end{array} \right.$$

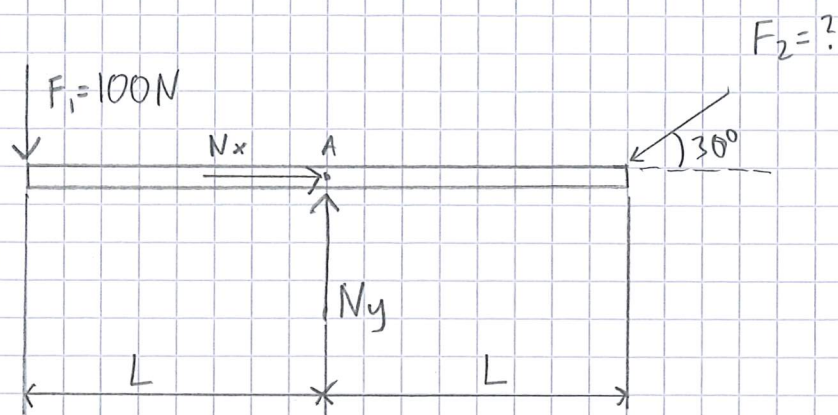
$$\text{II: } \left\{ \begin{array}{l} \curvearrow B: N_f \cdot (1650 + 990) - G \cdot 990 = 0 \end{array} \right.$$

$$\text{ekv II ger: } N_f = \frac{G \cdot 990}{1650 + 990} = 5625\text{ N}$$

$$\text{ekv I ger: } N_b = G - N_f = 15000 - 5625 = 9375\text{ N}$$

Svar: N_f är ca: 5630 N och N_b cirka 9380 N .

6



$$\begin{array}{l} \text{I} \\ \text{II} \\ \text{III} \end{array} \left\{ \begin{array}{l} \uparrow: N_y - F_1 - F_2 \cdot \sin(30^\circ) = 0 \\ \rightarrow: N_x - F_2 \cdot \cos(30^\circ) = 0 \\ \curvearrowright: F_2 \cdot \sin(30^\circ) \cdot L - F_1 \cdot L = 0 \end{array} \right.$$

Ekv. III ger storleken på F_2 :

$$F_2 = \frac{F_1 \cdot L}{\sin(30^\circ) \cdot L} = \frac{100}{\sin(30^\circ)} = \underline{\underline{200 \text{ N}}}$$

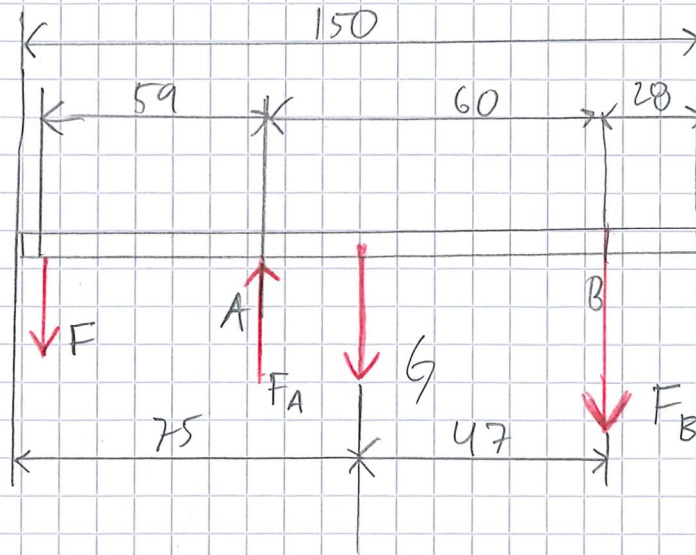
Ekv. II ger N_x :

$$N_x = F_2 \cdot \cos(30^\circ) = 200 \cdot \cos(30^\circ) \approx \underline{\underline{173,2 \text{ N}}}$$

Ekv. I ger N_y :

$$N_y = F_1 + F_2 \cdot \sin(30^\circ) = 100 + 200 \cdot \sin(30^\circ) = \underline{\underline{200 \text{ N}}}$$

7



$$G = 20 \text{ N}$$
$$F = 60 \text{ N}$$

$$\left\{ \begin{array}{l} \uparrow: F_A - G - F_B - F = 0 \\ \curvearrowright: F_A \cdot 60 - G \cdot 47 - F \cdot (59 + 60) = 0 \end{array} \right.$$

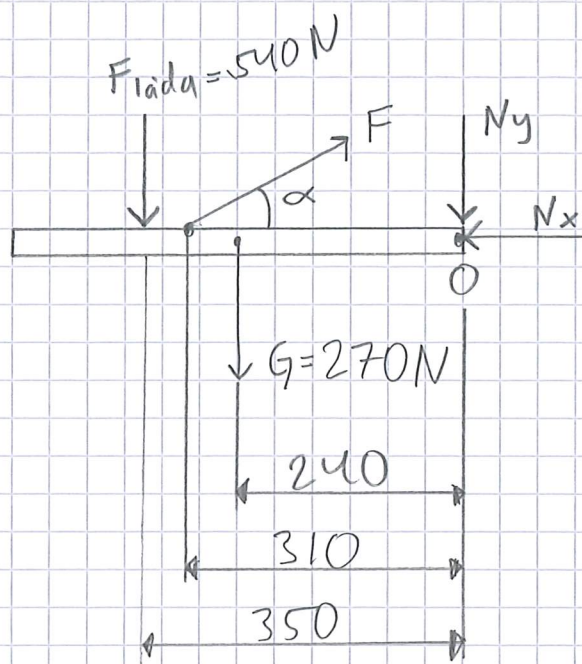
$$F_A = \frac{47 \cdot G + (59 + 60) \cdot F}{60} = \frac{47 \cdot 20 + (59 + 60) \cdot 60}{60} \approx 134,67 \text{ N}$$

$$F_B = F_A - G - F = 134,67 - 20 - 60 \approx 54,67 \text{ N}$$

Svar: F_A är cirka 135 N och F_B 55 N.

(mm)

8



Vinkeln α

är:

$$\alpha = \arctan\left(\frac{300}{310}\right) \approx$$

$$\approx 44,06^\circ$$

$$\begin{cases} \text{I} & \uparrow: F \cdot \sin(\alpha) - F_{\text{lada}} - G - N_y = 0 \\ \text{II} & \rightarrow: F \cdot \cos(\alpha) - N_x = 0 \\ \text{III} & \circlearrowleft: F \cdot \sin(\alpha) \cdot 310 - F_{\text{lada}} \cdot 350 - G \cdot 240 = 0 \end{cases}$$

ekv. III ger storleken på F .

$$F = \frac{F_{\text{lada}} \cdot 350 + G \cdot 240}{\sin(\alpha) \cdot 310} = \frac{540 \cdot 350 + 270 \cdot 240}{\sin(44,06^\circ) \cdot 310} \approx 1177 \text{ N}$$

Kraften i respektive rep är $\frac{F}{2} = \frac{1177}{2} \approx \underline{\underline{588,5 \text{ N}}}$

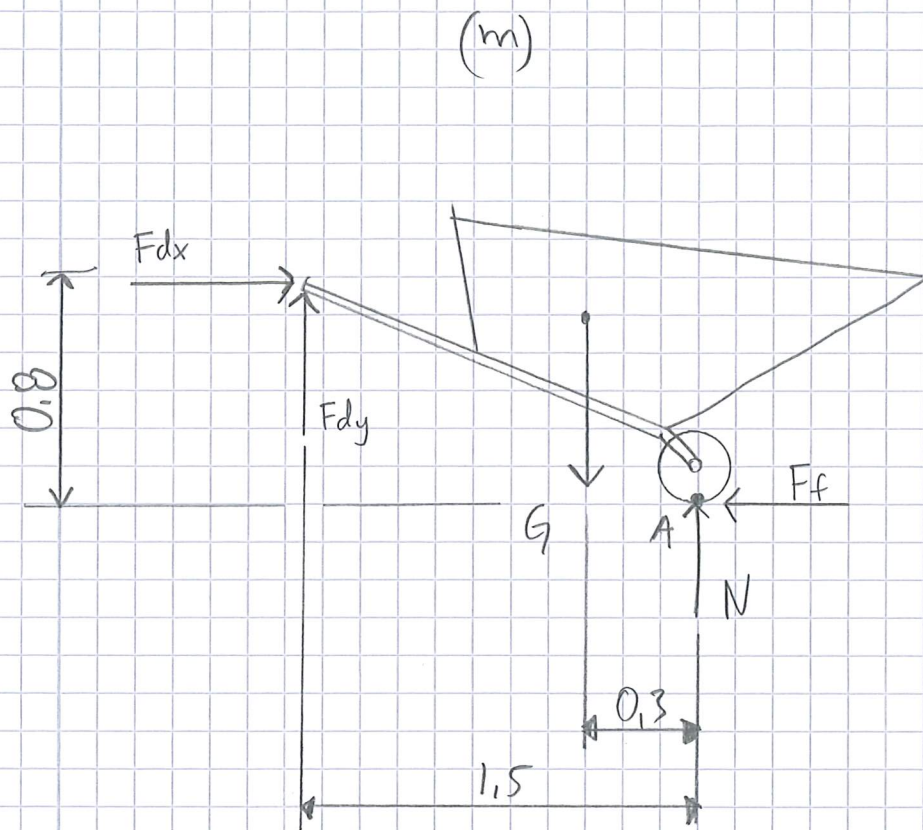
ekv II ger N_x :

$$N_x = F \cdot \cos(\alpha) = 1177 \cdot \cos(44,06^\circ) \approx \underline{\underline{846 \text{ N}}}$$

ekv I ger N_y :

$$N_y = F \cdot \sin(\alpha) - F_{\text{lada}} - G = 1177 \cdot \sin(44,06^\circ) - 540 - 270 \approx \underline{\underline{85 \text{ N}}}$$

9



$$G = 350 \text{ N}$$
$$\mu = 0,15$$

$$\begin{array}{l} \text{I} \\ \text{II} \\ \text{III} \end{array} \left\{ \begin{array}{l} \uparrow: N + F_{dy} - G = 0 \\ \rightarrow: F_{dx} - F_f = 0 \quad \text{där } F_f = \mu \cdot N \text{ dvs } F_{dx} - \mu \cdot N = 0 \\ \curvearrowright A: F_{dy} \cdot 1,5 + F_{dx} \cdot 0,8 - G \cdot 0,3 = 0 \end{array} \right.$$

stoppa in F_{dy} från ekv I och F_{dx} från ekv II i ekv III

$$\text{I: } F_{dy} = G - N$$

$$\text{II: } F_{dx} = \mu \cdot N$$

ger ekv III:

$$\overbrace{(G - N)}^{F_{dy}} \cdot 1,5 + \overbrace{\mu \cdot N}^{F_{dx}} \cdot 0,8 - G \cdot 0,3 = 0$$

$$1,5 G - 1,5 N + \mu \cdot N \cdot 0,8 = G \cdot 0,3$$

$$1,5 G - 0,3 G = 1,5 N - \mu \cdot N \cdot 0,8$$

1/2

9)

$$N = \frac{1,2 G}{1,5 - M \cdot 0,8} = \frac{1,2 \cdot 350}{1,5 - 0,15 \cdot 0,8} \approx 304,35 \text{ N}$$

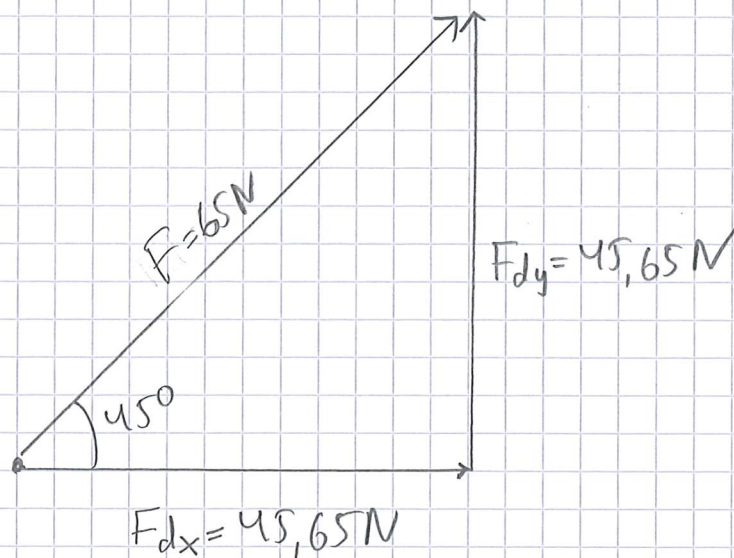
ekv II ger F_{dx} :

$$F_{dx} = M \cdot N = 0,15 \cdot 304,35 \approx 45,65 \text{ N}$$

ekv I ger F_{dy} :

$$F_{dy} = G - N = 350 - 304,35 = 45,65 \text{ N}$$

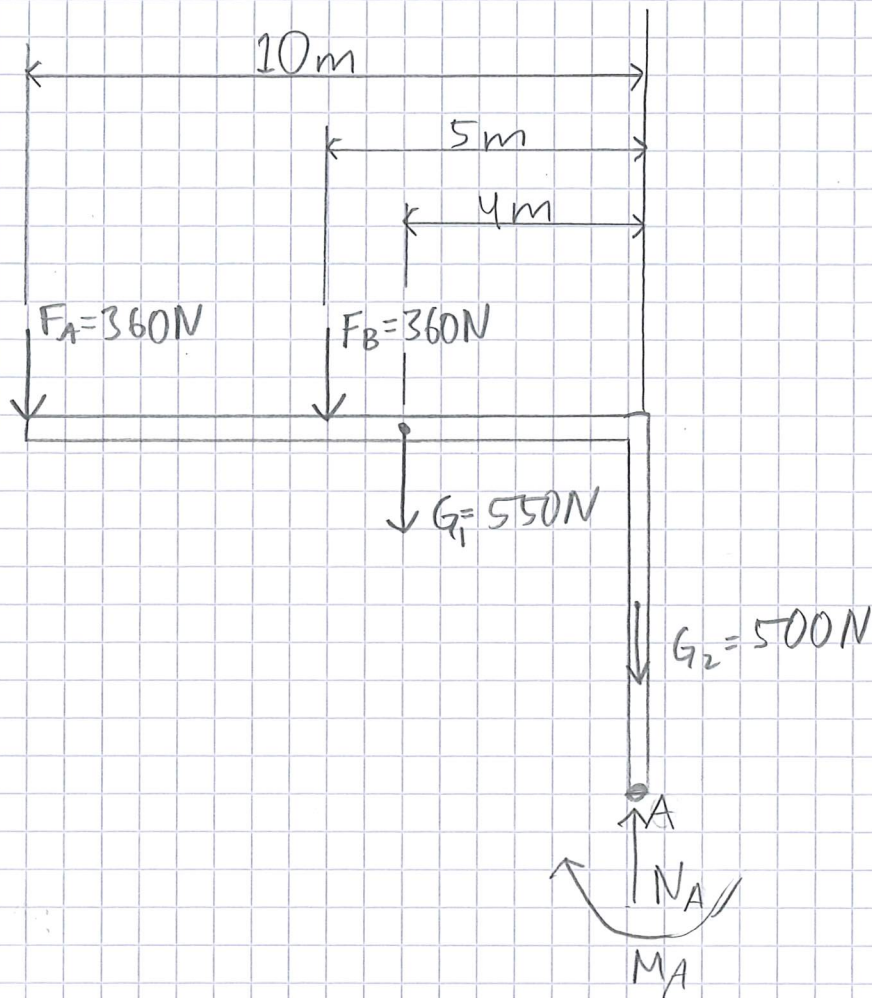
Totala krafterna F blir:



$$F = \sqrt{F_{dx}^2 + F_{dy}^2} = \sqrt{45,65^2 + 45,65^2} \approx \underline{\underline{65 \text{ N}}}$$

Svar: Kraften F blir cirka 65 N .

(10)



$$\begin{array}{l} \text{I} \\ \text{II} \end{array} \left\{ \begin{array}{l} \uparrow: N_A - G_1 - G_2 - F_A - F_B = 0 \\ \circlearrowleft: M_A - F_A \cdot 10 - F_B \cdot 5 - G_1 \cdot 4 = 0 \end{array} \right.$$

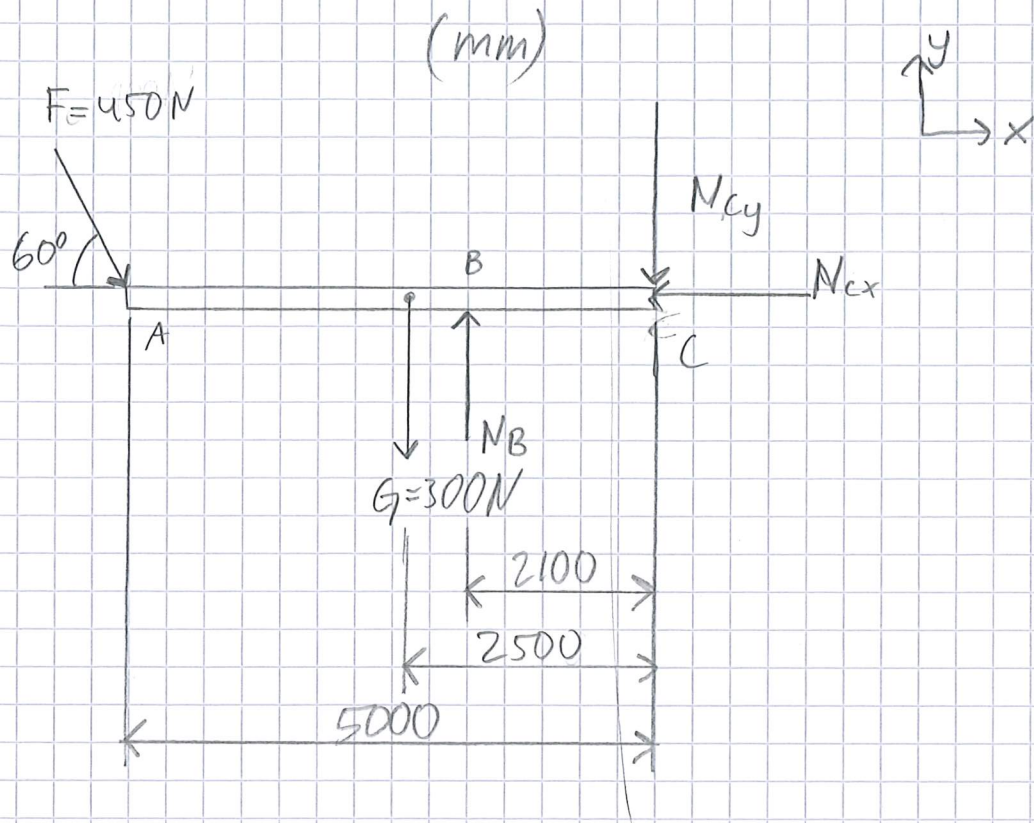
Ekv. I ger N_A :

$$N_A = G_1 + G_2 + F_A + F_B = 550 + 500 + 360 + 360 = \underline{\underline{1770\text{ N}}}$$

Ekv II ger M_A :

$$M_A = F_A \cdot 10 + F_B \cdot 5 + G_1 \cdot 4 = 360 \cdot 10 + 360 \cdot 5 + 550 \cdot 4 = \underline{\underline{7600\text{ Nm}}}$$

11



$$\begin{cases} \text{I} & \uparrow: N_B - G - N_{Cy} - F \cdot \sin(60^\circ) = 0 \\ \text{II} & \rightarrow: F \cdot \cos(60^\circ) - N_{Cx} = 0 \\ \text{III} & \curvearrowright: N_B \cdot 2100 - F \cdot \sin(60^\circ) \cdot 5000 - G \cdot 2500 = 0 \end{cases}$$

• Ekv. III ger N_B :

$$N_B = \frac{F \cdot \sin(60^\circ) \cdot 5000 + G \cdot 2500}{2100} = \frac{450 \cdot \sin(60^\circ) \cdot 5000 + 300 \cdot 2500}{2100} \approx 1285 \text{ N}$$

• Ekv. II ger N_{Cx} :

$$N_{Cx} = F \cdot \cos(60^\circ) = 450 \cdot \cos(60^\circ) = 225 \text{ N}$$

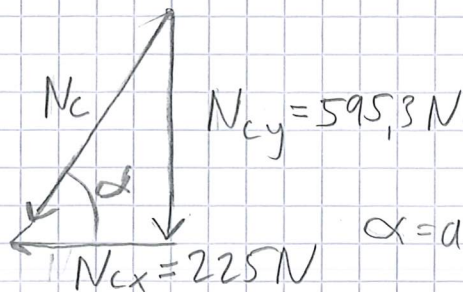
• Ekv. I ger N_{Cy} :

$$N_{Cy} = N_B - G - F \sin(60^\circ) = 1285 - 300 - 450 \cdot \sin(60^\circ) \approx 595,3 \text{ N}$$

N_A blir:

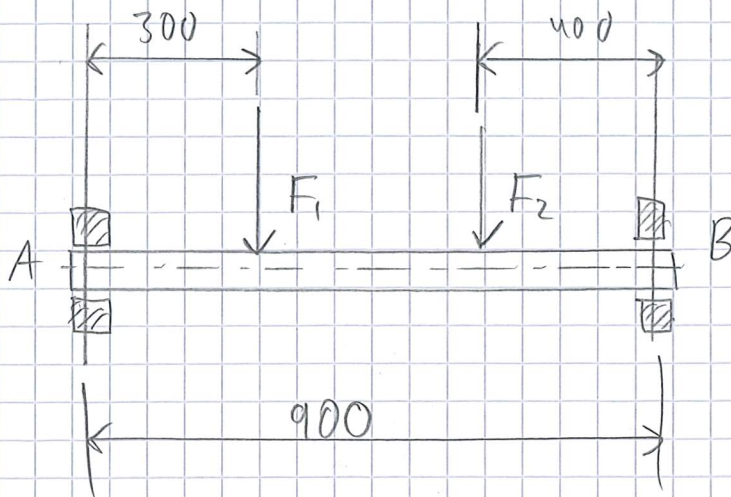
$$N_C = \sqrt{N_{Cy}^2 + N_{Cx}^2} = \sqrt{595,3^2 + 225^2} \approx 636,4 \text{ N}$$

Svar: N_B är cirka 1290 N och N_C cirka 636 N, med vinkeln $69,3^\circ$.

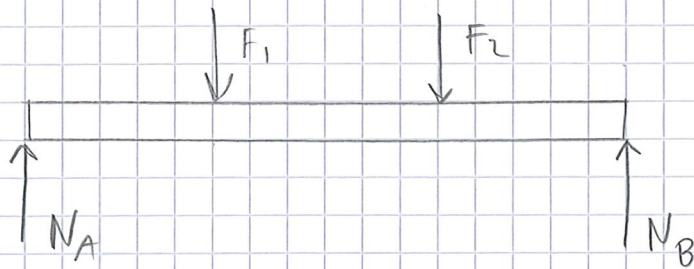


$$\alpha = \arctan\left(\frac{595,3}{225}\right) \approx 69,3^\circ$$

12



$$F_1 = 300 \text{ N}$$
$$F_2 = 450 \text{ N}$$

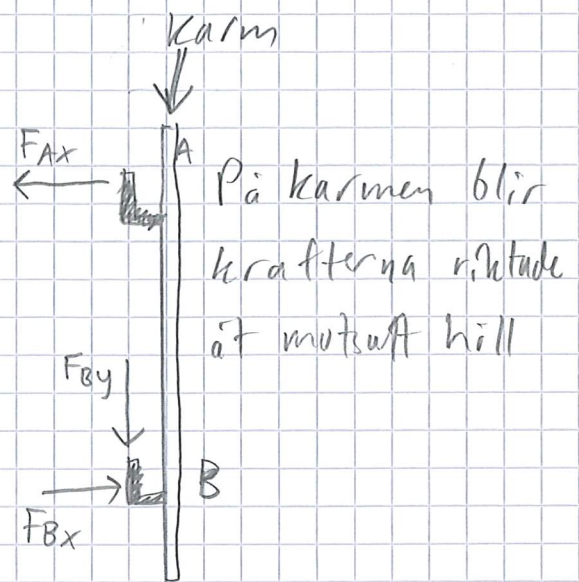
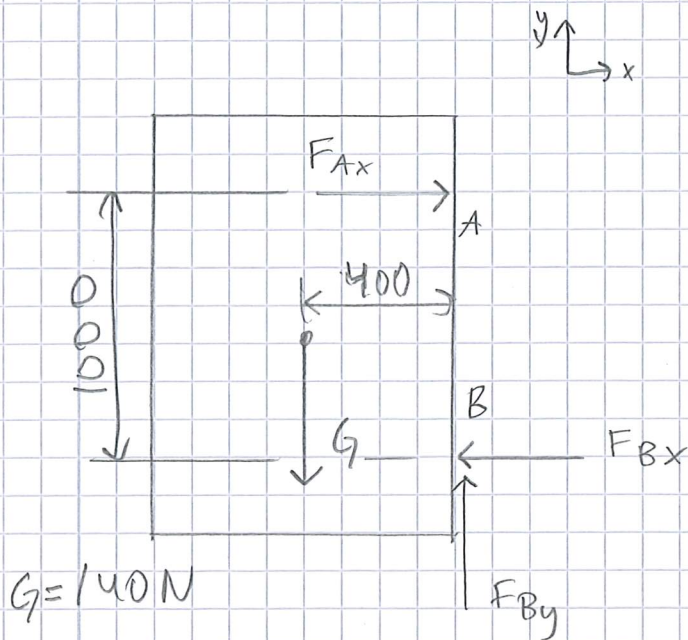


$$\begin{cases} \uparrow: N_A + N_B - F_1 - F_2 = 0 \\ \curvearrowright A: F_1 \cdot 300 + F_2 \cdot 500 - N_B \cdot 900 = 0 \end{cases}$$

$$\Rightarrow N_B = \frac{F_1 \cdot 300 + F_2 \cdot 500}{900} = \frac{300 \cdot 300 + 450 \cdot 500}{900} = 350 \text{ N}$$

$$\Rightarrow N_A = F_1 + F_2 - N_B = 300 + 450 - 350 = 400 \text{ N}$$

13



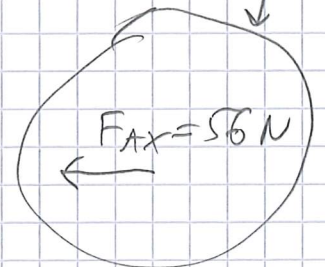
$$\begin{aligned}
 \text{I} & \left\{ \begin{array}{l} \uparrow: F_{By} - G = 0 \\ \rightarrow: F_{Ax} - F_{Bx} = 0 \\ \curvearrowright: F_{Ax} \cdot 1000 - G \cdot 400 = 0 \end{array} \right.
 \end{aligned}$$

$$\text{I} \Rightarrow F_{By} = G = 140 \text{ N}$$

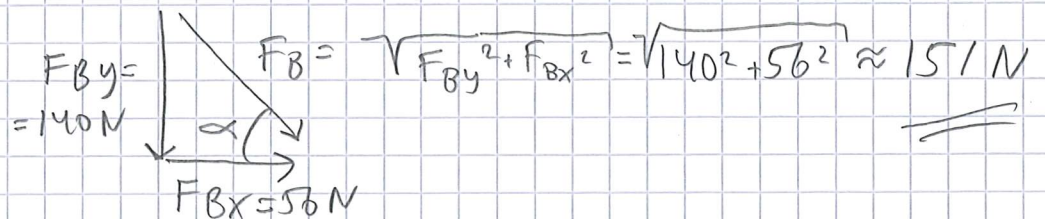
$$\text{III} \Rightarrow F_{Ax} = \frac{G \cdot 400}{1000} = \frac{140 \cdot 400}{1000} = 56 \text{ N}$$

$$\text{II} \Rightarrow F_{Bx} = F_{Ax} = 56 \text{ N}$$

Med avseende på karmen

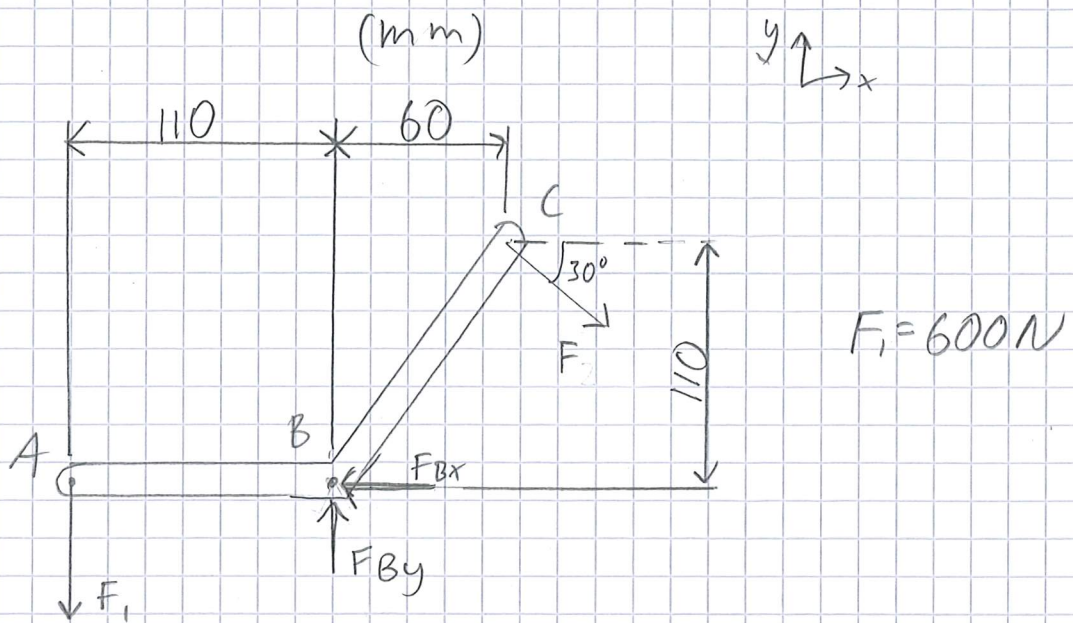


Resultant vid B (med avseende på karmen)



$$\alpha = \arctan\left(\frac{140}{56}\right) \approx 68,2^\circ$$

(14)



$$\begin{array}{l} \text{I} \\ \text{II} \\ \text{III} \end{array} \left\{ \begin{array}{l} \uparrow: F_{By} - F_1 - F \cdot \sin(30^\circ) = 0 \\ \rightarrow: F \cdot \cos(30^\circ) - F_{Bx} = 0 \\ \curvearrowright B: F \cdot \cos(30^\circ) \cdot 110 - F_1 \cdot 110 + F \cdot \sin(30^\circ) \cdot 60 = 0 \end{array} \right.$$

Tre ekvationer och tre obekanta, bra!

ekv III ger F :

$$F = \frac{F_1 \cdot 110}{110 \cdot \cos(30^\circ) + 60 \cdot \sin(30^\circ)} = \frac{600 \cdot 110}{110 \cdot \cos(30^\circ) + 60 \cdot \sin(30^\circ)} \approx \underline{\underline{526,9 \text{ N}}}$$

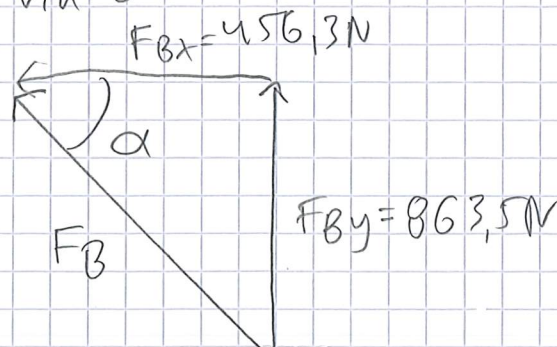
ekv II ger F_{Bx} :

$$F_{Bx} = F \cdot \cos(30^\circ) = 526,9 \cdot \cos(30^\circ) \approx 456,3 \text{ N}$$

ekv I ger F_{By} :

$$F_{By} = F_1 + F \cdot \sin(30^\circ) = 600 + 526,9 \cdot \sin(30^\circ) \approx \underline{\underline{863,5 \text{ N}}}$$

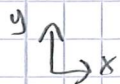
kraften vid B



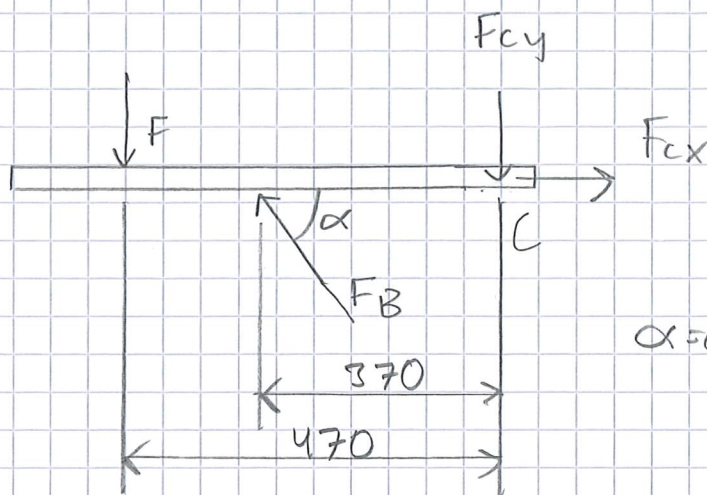
$$\alpha = \arctan\left(\frac{863,5}{456,3}\right) \approx \underline{\underline{62,15^\circ}}$$

$$\begin{aligned} F_B &= \sqrt{F_{Bx}^2 + F_{By}^2} = \\ &= \sqrt{456,3^2 + 863,5^2} \approx \\ &\approx \underline{\underline{976,7 \text{ N}}} \end{aligned}$$

(15)



$$F = 300 \text{ N}$$



$$\alpha = \arctan\left(\frac{300}{400}\right) \approx 36,9^\circ$$

$$\begin{array}{l} \text{I} \\ \text{II} \\ \text{III} \end{array} \left\{ \begin{array}{l} \uparrow: F_B \cdot \sin(\alpha) - F - F_{cy} = 0 \\ \rightarrow: F_{cx} - F_B \cdot \cos(\alpha) = 0 \\ \curvearrowright_C: F_B \cdot \sin(\alpha) \cdot 370 - F \cdot 470 = 0 \end{array} \right.$$

ekv III ger F_B :

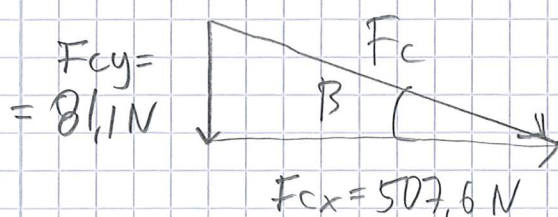
$$F_B = \frac{F \cdot 470}{\sin(\alpha) \cdot 370} = \frac{300 \cdot 470}{\sin(36,9^\circ) \cdot 370} \approx \underline{\underline{634,7 \text{ N}}}$$

ekv II ger F_{cx} :

$$F_{cx} = F_B \cdot \cos(\alpha) = 634,7 \cdot \cos(36,9^\circ) \approx 507,6 \text{ N}$$

ekv I ger F_{cy}

$$F_{cy} = F_B \cdot \sin(\alpha) - F = 634,7 \cdot \sin(36,9^\circ) - 300 \approx 81,1 \text{ N}$$



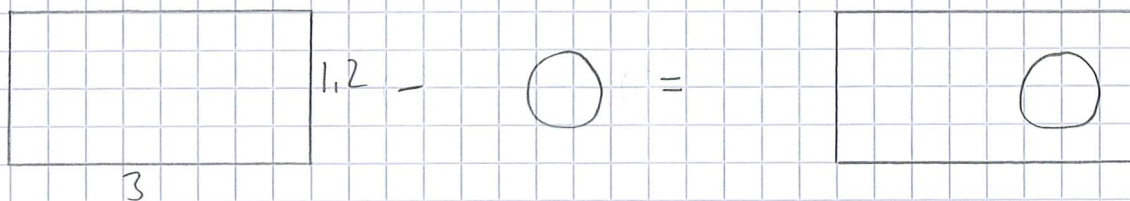
$$\beta = \arctan\left(\frac{81,1}{507,6}\right) \approx \underline{\underline{9,1^\circ}}$$

$$\begin{aligned} F_c &= \sqrt{F_{cy}^2 + F_{cx}^2} = \\ &= \sqrt{81,1^2 + 507,6^2} \approx \underline{\underline{514 \text{ N}}} \end{aligned}$$

16

$$G = 120 \cdot g = 1200 \text{ N}$$

(m)



$$A_1 = 3 \cdot 1,2 = 3,6 \text{ m}^2$$

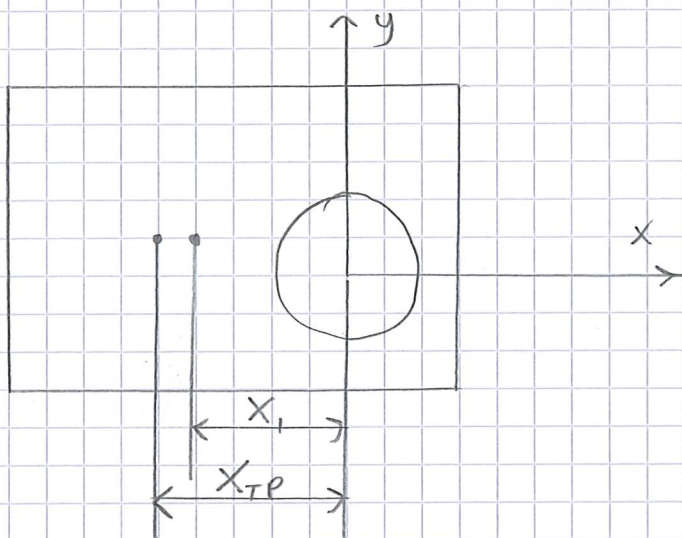
$$x_1 = 0,8 \text{ m}$$

$$A_2 = \frac{\pi \cdot 0,8^2}{4} = \frac{4\pi}{25} \text{ m}^2$$

$$x_2 = 0 \text{ m}$$

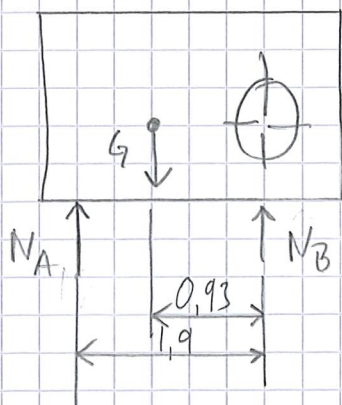
$$A = 3,6 - \frac{4\pi}{25} \text{ m}^2$$

$$x_{TP} = ?$$



$$x_{TP} = \frac{A_1 \cdot x_1 - A_2 \cdot x_2}{A_1 - A_2} = \frac{3,6 \cdot 0,8 - \frac{4\pi}{25} \cdot 0}{3,6 - \frac{4\pi}{25}} \approx 0,93 \text{ m}$$

(m)



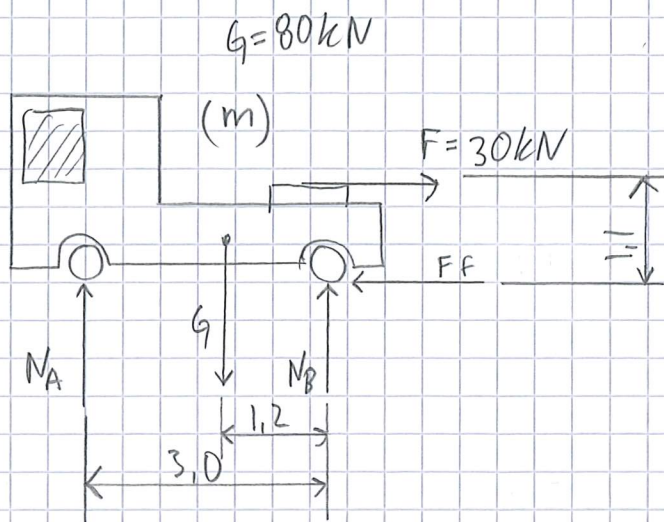
$$\begin{cases} \uparrow: N_A + N_B - G = 0 \\ \curvearrowright: N_A \cdot 1,9 - G \cdot 0,93 = 0 \end{cases}$$

$$\Rightarrow N_A = \frac{G \cdot 0,93}{1,9} = \frac{1200 \cdot 0,93}{1,9} \approx 587,4 \text{ N}$$

$$\Rightarrow N_B = G - N_A = 1200 - 587,4 \approx 612,6 \text{ N}$$

Svar: krafterna N_A är cirka 587 N och N_B cirka 613 N.

17



$$\begin{array}{l} \text{I} \\ \text{II} \\ \text{III} \end{array} \left\{ \begin{array}{l} \uparrow: N_A + N_B - G = 0 \\ \rightarrow: F - F_f = 0 \\ \curvearrowright: N_A \cdot 3,0 - G \cdot 1,2 + F \cdot 1,1 = 0 \end{array} \right.$$

ekv III ger N_A :

$$N_A = \frac{G \cdot 1,2 - F \cdot 1,1}{3,0} = \frac{80\,000 \cdot 1,2 - 30\,000 \cdot 1,1}{3,0} = 21\,000 \text{ kN}$$

ekv I ger N_B :

$$N_B = G - N_A = 80\,000 - 21\,000 = 59\,000 \text{ kN}$$

ekv II ger F_f :

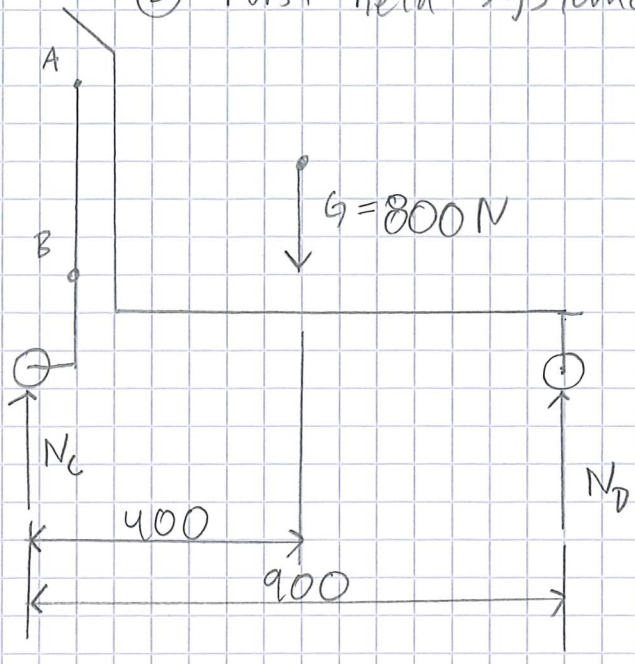
$$F_f = F = 30\,000 \text{ kN}$$

Frictionstal $F_f = N_B \cdot \mu \Rightarrow \mu = \frac{F_f}{N_B} = \frac{30}{59} \approx 0,51$

Svar: ett frictionstal på cirka 0,5 krävs.

10

① Först hela systemet, ger N_c och N_D .



$$\begin{cases} \uparrow: N_c + N_D - G = 0 \\ \curvearrowright: G \cdot 400 - N_D \cdot 900 = 0 \end{cases}$$

$$\Rightarrow N_D = \frac{G \cdot 400}{900} = \frac{800 \cdot 400}{900} \approx 355,6 \text{ N}$$

$$\Rightarrow N_C = G - N_D = 800 - 355,6 \approx 444,4 \text{ N}$$

② Frlägg delar för resterande krafter, ta bort balken vid hjulet C.

(mm)

$$\begin{cases} \uparrow: N_D - G + F_{Ay} = 0 \\ \rightarrow: F_{Ax} - F_B = 0 \\ \curvearrowright: F_{Ax} \cdot (180 + 420) - G \cdot 500 + F_{Ay} \cdot (900 - 90) - F_B \cdot 180 = 0 \end{cases}$$

ersätt balken med krafter...

kraft och motkraft förklarar krafternas riktningar här

Krafterna F_{Ay} & N_c ger medursrotation \Rightarrow därför blir F_B och F_{Ax} riktade åt detta håll.

Frläggning balk AC.

18

ekv I ger F_{Ay}

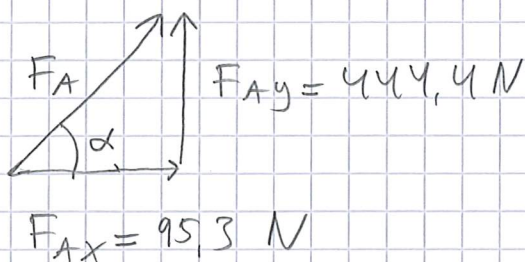
$$\Rightarrow F_{Ay} = G - N_D = 800 - 355,6 = 444,4 \text{ N}$$

ekv II ger: $F_{Ax} = F_B$

ekv III ger F_{Ax} och F_B :

$$F_{Ax}(180+420) - G \cdot 500 + F_{Ay}(900-90) - F_B \cdot 180 = 0$$

$$F_B = F_{Ax} = \frac{G \cdot 500 - F_{Ay} \cdot (900-90)}{(180+420) - 180} = \frac{800 \cdot 500 - 444,4 \cdot (900-90)}{420} \approx 95,3 \text{ N}$$

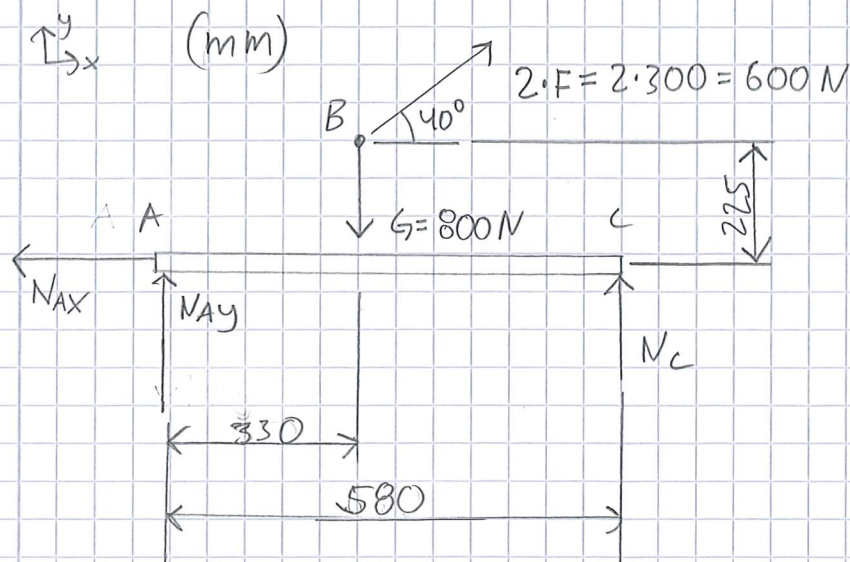


$$F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} = \sqrt{95,3^2 + 444,4^2} \approx 454,5 \text{ N}$$

$$\alpha = \arctan\left(\frac{444,4}{95,3}\right) \approx 77,9^\circ$$

Svar: Vid C blir N_C cirka 444 N, vid D blir N_D cirka 356 N. Kraften vid A är cirka 455 N med vinkeln 78° och vid B är F_B cirka 95 N.

(19)



$$\begin{array}{l} \text{I} \\ \text{II} \\ \text{III} \end{array} \left\{ \begin{array}{l} \uparrow: N_C + N_{Ay} - G + 2F \cdot \sin(40^\circ) = 0 \\ \rightarrow: 2F \cdot \cos(40^\circ) - N_{Ax} = 0 \\ \vec{A}: G \cdot 330 - N_C \cdot 580 + 2F \cdot \cos(40^\circ) \cdot 225 - 2F \sin(40^\circ) \cdot 330 = 0 \end{array} \right.$$

ekv III ger N_C :

$$\begin{aligned} N_C &= \frac{G \cdot 330 + 2F \cos(40^\circ) \cdot 225 - 2F \cdot \sin(40^\circ) \cdot 330}{580} \\ &= \frac{800 \cdot 330 + 2 \cdot 300 \cdot \cos(40^\circ) \cdot 225 - 2 \cdot 300 \cdot \sin(40^\circ) \cdot 330}{580} \approx 414 \text{ N} \end{aligned}$$

ekv II ger N_{Ax} :

$$N_{Ax} = 2F \cdot \cos(40^\circ) = 2 \cdot 300 \cdot \cos(40^\circ) \approx 459,6 \text{ N}$$

ekv I ger N_{Ay} :

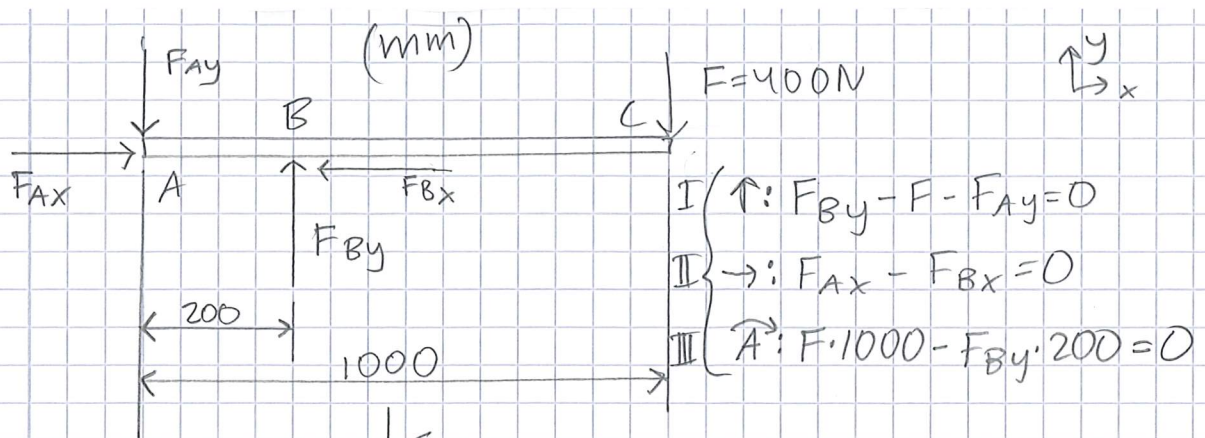
$$N_{Ay} = G - 2F \sin(40^\circ) - N_C = 800 - 2 \cdot 300 \cdot \sin(40^\circ) - 414 \approx 0,3 \text{ N}$$

$$\begin{array}{c} N_{Ax} = 459,6 \text{ N} \\ \leftarrow N_A \nearrow \\ N_{Ay} = 0,3 \text{ N} \end{array}$$

$$\alpha = \arctan\left(\frac{0,3}{459,6}\right) \approx 0,04^\circ$$

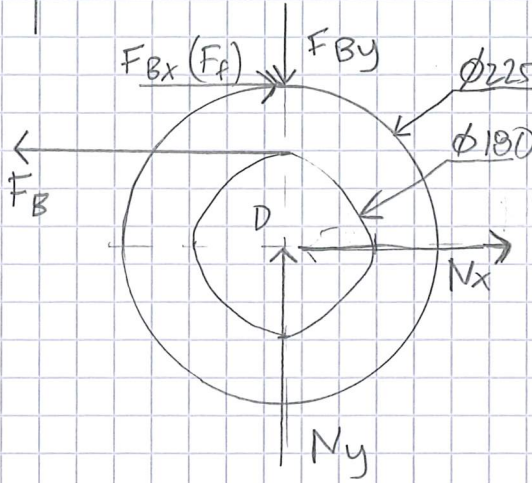
$$N_A = \sqrt{N_{Ay}^2 + N_{Ax}^2} = \sqrt{0,3^2 + 459,6^2} \approx 459,6 \text{ N}$$

20



$$\begin{aligned} \text{I} \uparrow: & F_{By} - F - F_{Ay} = 0 \\ \text{II} \rightarrow: & F_{Ax} - F_{Bx} = 0 \\ \text{III} \curvearrowright: & F \cdot 1000 - F_{By} \cdot 200 = 0 \end{aligned}$$

$$M = 0,3$$



$$\begin{aligned} \text{IV} \uparrow: & N_y - F_{By} = 0 \\ \text{V} \rightarrow: & F_{Bx} - F_B + N_x = 0 \\ \text{VI} \curvearrowright: & F_{Bx} \cdot \frac{225}{2} - F_B \cdot \frac{180}{2} = 0 \end{aligned}$$

$$\text{VII} \quad F_{Bx} = F_{By} \cdot M$$

ekv III ger F_{By} :

$$F_{By} = \frac{F \cdot 1000}{200} = \frac{400 \cdot 1000}{200} = 2000 \text{ N}$$

ekv VII ger F_{Bx} :

$$F_{Bx} = F_{By} \cdot M = 2000 \cdot 0,3 = 600 \text{ N}$$

ekv VI ger F_B :

$$F_B = \frac{F_{Bx} \cdot \frac{225}{2}}{\frac{180}{2}} = \frac{600 \cdot \frac{225}{2}}{\frac{180}{2}} = \underline{\underline{750 \text{ N}}}$$