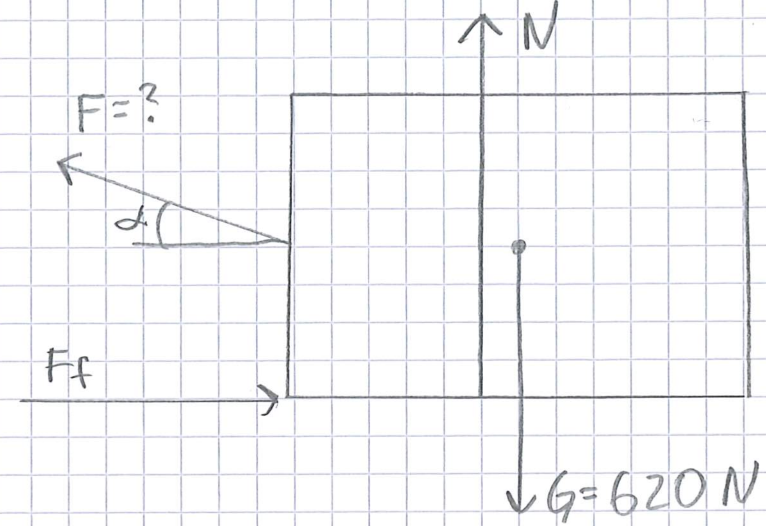
⑤




där $\mu=0,4$

$$\alpha=\arctan\left(\frac{0,40}{1,5}\right)\approx 14,93^\circ$$

$$\begin{cases} \text{I: } \uparrow: N-G+F\cdot\sin(14,93^\circ)=0 \\ \text{II: } \rightarrow: F_f-F\cdot\cos(14,93^\circ)=0 \end{cases} \quad (\text{där } F_f=\mu\cdot N)$$

ekvation II ger N:

$$\overbrace{\mu\cdot N}^{F_f}-F\cdot\cos(14,93^\circ)=0$$

$$N=\frac{F\cdot\cos(14,93^\circ)}{\mu}$$

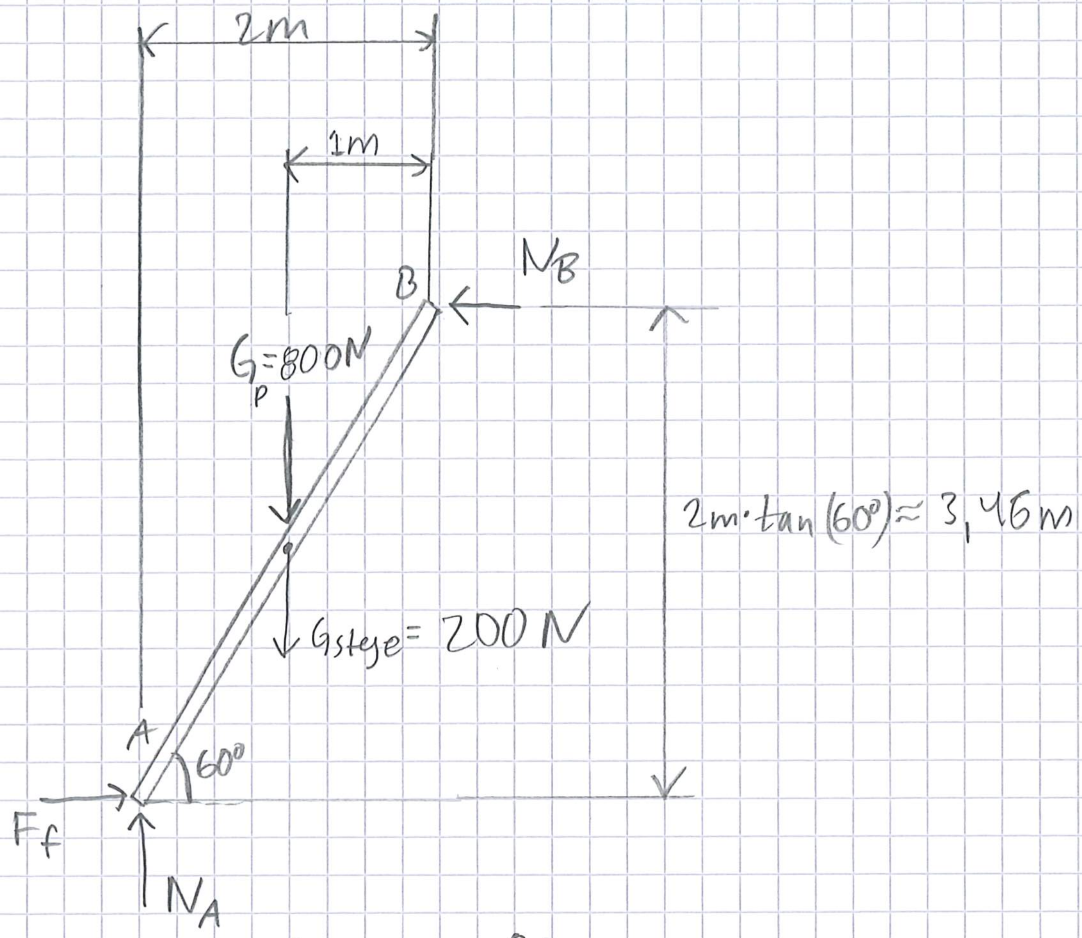
stoppa in detta i ekvation I:

$$\overbrace{\frac{F\cdot\cos(14,93^\circ)}{\mu}}^{N}-G+F\sin(14,93^\circ)=0$$

$$F=\frac{G}{\frac{\cos(14,93^\circ)}{\mu}+\sin(14,93^\circ)}=\frac{620}{\frac{\cos(14,93^\circ)}{0,4}+\sin(14,93^\circ)}\approx$$

$$\approx \underline{\underline{232\,N}}$$

⑥




Hur stor friktionskraft $F_f$ krävs för att stegen inte ska börja glida?

$\uparrow: N_A - G_{stege} - G_p = 0$

$\Rightarrow N_A = G_{stege} + G_p = 200 + 800 = 1000N$

$\rightarrow: F_f - N_B = 0$

$\Rightarrow F_f = N_B$

Här behövs ytterligare en ekvation.

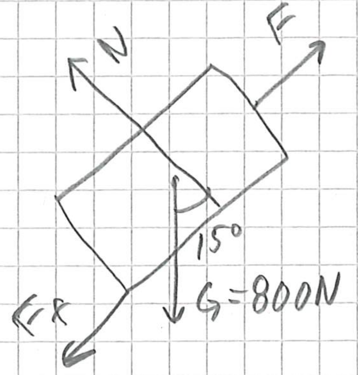
$\overset{\curvearrowright}{A}: (G_p + G_{stege}) \cdot 1 - N_B \cdot 3,46 = 0$

$\Rightarrow N_B = \frac{G_p + G_{stege}}{3,46} = \frac{1000}{3,46} \approx 289N$

$\underline{\underline{F_f = N_B = 289N}}$

Det krävs en friktionskraft på minst 290N...

7

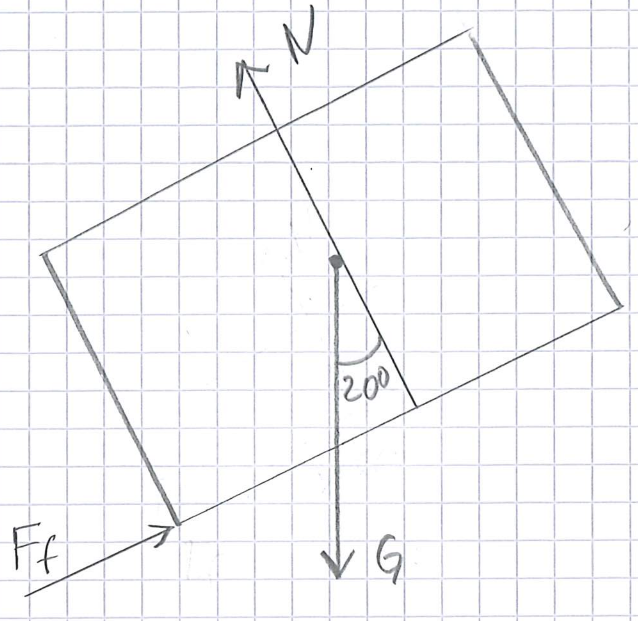



$\mu = 0{,}176$

$$\begin{cases} \nearrow: F - F_f - G\sin(15^\circ) = 0 & \Rightarrow F = \mu \cdot N + G\sin(15^\circ) \\ \nwarrow: N - G\cdot\cos(15^\circ) = 0 & \Rightarrow N = G\cdot\cos(15^\circ) \end{cases}$$

$$F = 0{,}176 \cdot \overbrace{800\cos(15^\circ)}^{N} + 800\cdot\sin(15^\circ) \approx \underline{\underline{343{,}3\ N}}$$

8




$$\begin{cases} \nearrow: F_f - G \cdot \sin(20°) = 0 & \text{I} \\ \nwarrow: N - G \cdot \cos(20°) = 0 & \text{II} \end{cases}$$

där $F_f = \mu \cdot N$

ekvation II ger: $N = G \cdot \cos(20°)$

ekvation I är

$$F_f - G \cdot \sin(20°) = 0$$

$$\mu \cdot N - G \cdot \sin(20°) = 0$$

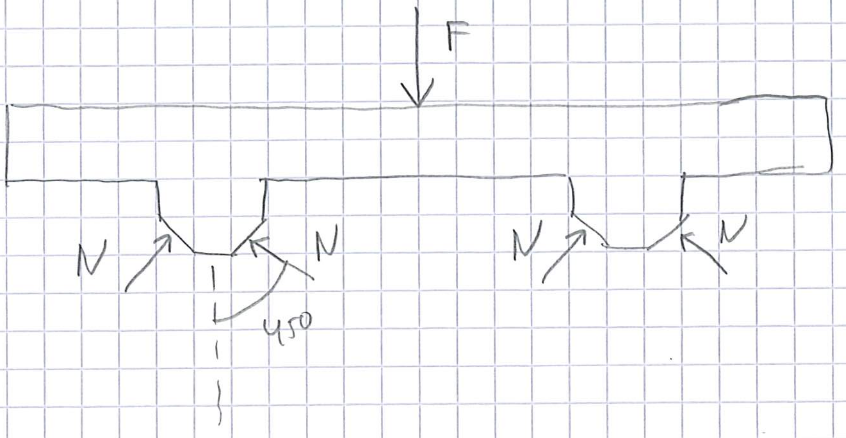
$$\mu \cdot G \cdot \cos(20°) - G \cdot \sin(20°) = 0$$

$$\mu = \frac{\cancel{G} \cdot \sin(20°)}{\cancel{G} \cdot \cos(20°)} = \tan(20°)$$

säkerhet 1,5 $\Rightarrow$

$$\mu = 1,5 \cdot \tan(20°) \approx \underline{\underline{0,55}}$$

⑨ Frilägg balken




$\uparrow: 4 \cdot N \cdot \cos(45°) - F = 0$

$\Rightarrow N = \dfrac{F}{4 \cdot \cos(45°)}$

$F = 12\ kN$

$\mu = 0{,}1$

Friktionskraften vid varje kontaktyta blir:

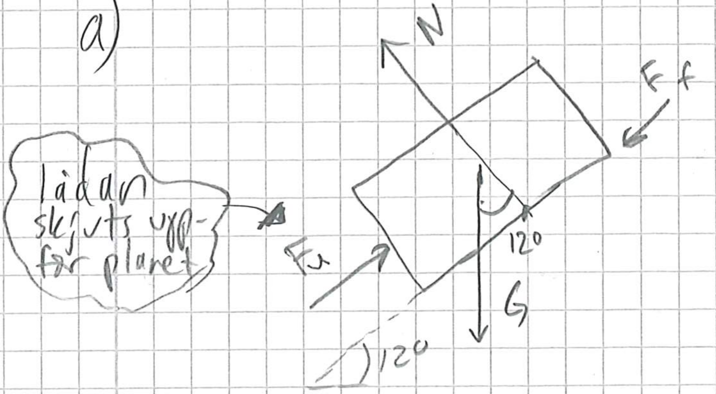
$F_f = \mu \cdot N = \mu \cdot \dfrac{F}{4 \cdot \cos(45°)} = 0{,}1 \cdot \dfrac{12000}{4 \cdot \cos(45°)} \approx 424{,}3\ N$

Totala friktionskraften blir

$F_f \cdot (\text{fyra kontaktytor}) = F_f \cdot 4 = 424{,}3 \cdot 4 \approx \underline{\underline{1{,}7\ kN}}$

10

a)




$\mu = 0,25$

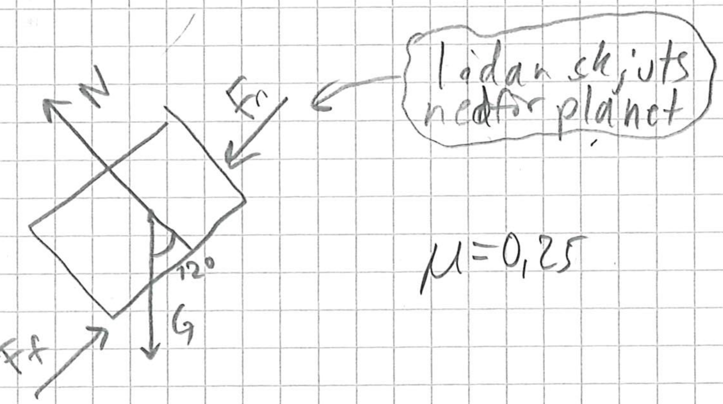
$\nearrow: F_u - F_f - G\sin(12°) = 0 = F_u - \mu \cdot N - G\sin(12°)$

$\nwarrow: N - G \cdot \cos(12°) = 0 \implies N = G \cdot \cos(12°)$

$F_u = \mu \cdot G \cdot \cos(12°) + G\sin(12°) =$

$= 0,25 \cdot 800\cos(12°) + 800 \cdot \sin(12°) \approx \underline{\underline{362\ N}}$

b)



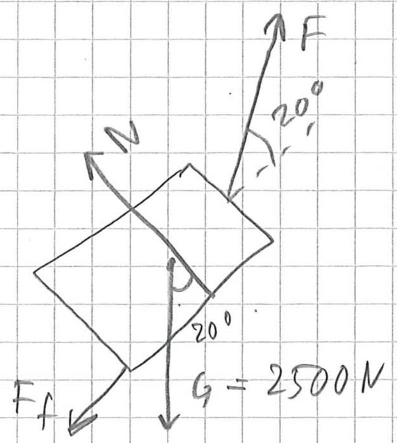

$\mu = 0,25$

$\nearrow: F_f - F_n - G\sin(12°) = 0 = \mu \cdot N - F_n - G\sin(12°)$

$\nwarrow: N - G\cos(12°) = 0 \implies N = G\cos(12°)$

$F_n = 0,25 \cdot 800\cos(12°) - 800\sin(12°) \approx \underline{\underline{29,3\ N}}$

(11)




$\mu = 0,1$

$$\begin{cases} \text{I: } \nearrow: F\cdot\cos(20°) - F_f - G\cdot\sin(20°) = 0 \\ \text{II: } \nwarrow: N - G\cdot\cos(20°) + F\sin(20°) = 0 \end{cases}$$

(där $F_f = \mu \cdot N$)

ekv II: $\Rightarrow N = G\cdot\cos(20°) - F\sin(20°)$

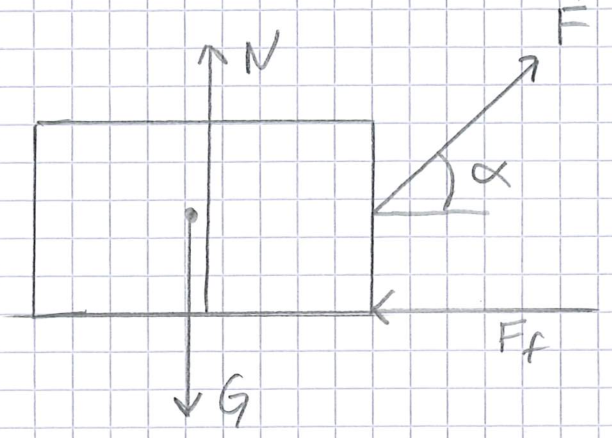
ekv I: $\Rightarrow F\cos(20°) - \mu\cdot[G\cos(20°) - F\sin(20°)] - G\sin(20°) = 0$

$$F[\cos(20°) + \mu\cdot\sin(20°)] = G\sin(20°) + \mu G\cos(20°)$$

$$F = \frac{G\sin(20°) + \mu G\cos(20°)}{\cos(20°) + \mu\sin(20°)} = \frac{2500\cdot\sin(20°) + 0{,}1\cdot 2500\cdot\cos(20°)}{\cos(20°) + 0{,}1\cdot\sin(20°)} \approx$$

$\approx \underline{\underline{1120\text{ N}}}$

(13) Vilken vinkel $\alpha$ kräver minst kraft $(F)$?




$\mu = 0,6$

$$
\begin{cases}
\text{I} \quad \uparrow: N - G + F \cdot \sin(\alpha) = 0 \\
\text{II} \quad \rightarrow: F \cdot \cos(\alpha) - F_f = 0 \\
\text{III} \quad F_f = \mu \cdot N
\end{cases}
$$

• Ekv I ger N:

$$N = G - F\sin(\alpha)$$

• Ekv III i II ger N (på annat sätt):

$$F \cdot \cos(\alpha) - F_f = 0$$

$$F \cdot \cos(\alpha) - \mu \cdot N = 0$$

$$\Rightarrow N = \frac{F \cdot \cos(\alpha)}{\mu}$$

$N = N$ (samma kraft)

$$G - F\sin(\alpha) = \frac{F \cdot \cos(\alpha)}{\mu}$$

$$\Rightarrow F(\alpha) = \frac{G}{\sin(\alpha) + \frac{\cos(\alpha)}{\mu}}$$

13

Vi skulle kunna derivera funktionen $F(\alpha)$ och sätta derivatan lika med noll. Men vi kan också "plotta" funktionen i en graf.

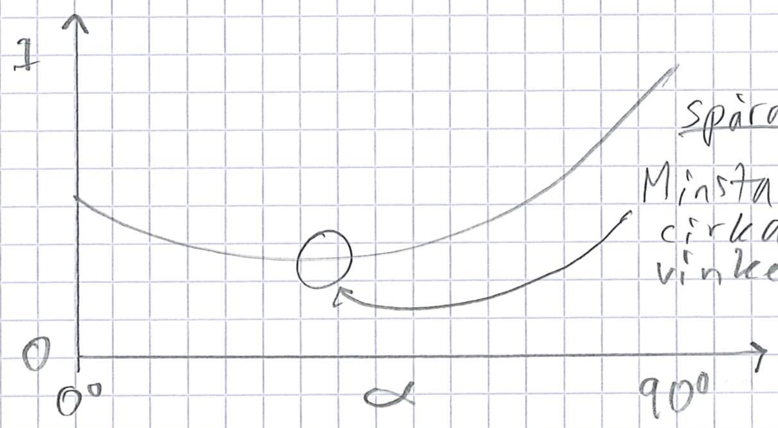
Sätt $G=1$ och skriv i miniräknaren:

$$y_1 = \frac{1}{\sin(x) + \frac{\cos(x)}{0,6}}$$

Rita en graf med gränsvärden:

$$\begin{cases} 0 < x < 90 \text{ (grader)} \\ 0 < y < 1 \end{cases}$$

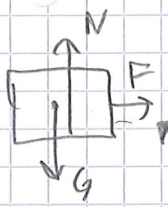
Obs! välj grader och inte radianer.




Spåra minimum

Minsta värdet blir cirka $0,51 \cdot G$ vid vinkeln $\alpha = 30,96°$.

Svar: Det behövs cirka $0,51 \cdot G$ N vid vinkeln cirka 31°.



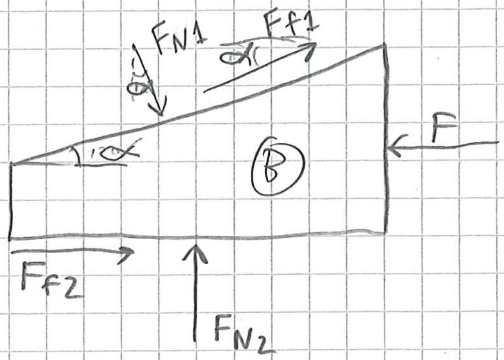

Om $\alpha = 0° \Rightarrow F(0°) = \frac{G}{\sin(0) + \frac{\cos(0)}{\mu}} = G \cdot \mu$

vilket stämmer bra då $F_f = N \cdot \mu = G \cdot \mu$.

Om $\alpha = 90° \Rightarrow F(90°) = \frac{G}{\sin(90°) + \frac{\cos(90°)}{\mu}} = G$


vilket stämmer bra eftersom lådan bärs!

(14)




$$\text{I}\quad \uparrow: F_{N2} - F_{N1}\cdot \cos(\alpha) + F_{f1}\cdot \sin(\alpha) = 0$$

$$\text{I}\quad \uparrow: F_{N2} - F_{N1} \cos(\alpha) + \mu F_{N1} \sin(\alpha) = 0$$

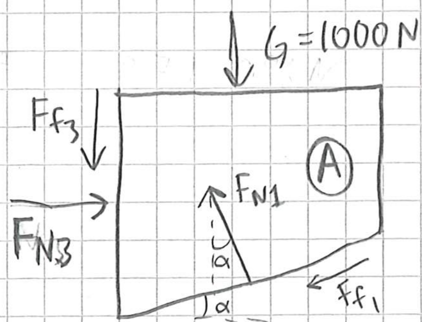
$$\text{II}\quad \rightarrow: F_{f2} + F_{f1} \cos(\alpha) + F_{N1} \sin(\alpha) - F = 0$$

$$\text{II}\quad F_{N2}\cdot \mu + F_{N1}\cdot \mu \cos(\alpha) + F_{N1} \sin(\alpha) - F = 0$$

$$\text{I}\quad F_{N2} = F_{N1}\left(\cos(\alpha) - \mu \sin(\alpha)\right)$$

$$\text{II}\quad \left[F_{N1}(\cos(\alpha) - \mu \sin(\alpha))\right]\cdot \mu + F_{N1}\cdot \mu \cos(\alpha) + F_{N1} \sin(\alpha) = F$$

$$\text{V}\quad F_{N1}\left[\mu \cos(\alpha) - \mu^2 \sin(\alpha) + \mu \cos(\alpha) + \sin(\alpha)\right] = F$$




$$\text{III}\quad \uparrow: F_{N1} \cos(\alpha) - F_{f3} - G - F_{f1}\cdot \sin(\alpha) = 0$$

$$F_{N1} \cos(\alpha) - F_{N3}\cdot \mu - G - F_{N1}\cdot \mu \sin(\alpha) = 0$$

$$\text{IV}\quad \rightarrow: F_{N3} - F_{N1} \sin(\alpha) - F_{f1}\cdot \cos(\alpha) = 0$$

$$F_{N3} - F_{N1} \sin(\alpha) - F_{N1} \mu \cdot \cos(\alpha) = 0$$

$$F_{N3} = F_{N1}\left(\sin(\alpha) + \mu \cos(\alpha)\right)$$

(14)

IV i III

$$F_{N1} \cdot \cos(\alpha) - \left[F_{N1}\left(\sin(\alpha) + \mu\cos(\alpha)\right)\right] \cdot \mu - G - F_{N1} \cdot \mu\sin(\alpha) = 0$$

VI $$F_{N1}\left[\cos(\alpha) - \mu\sin(\alpha) - \mu^2\cos(\alpha) - \mu\sin(\alpha)\right] = G$$

VI i V $$\frac{G \cdot \left[\mu \cdot \cos(\alpha) - \mu^2\sin(\alpha) + \mu\cos(\alpha) + \sin(\alpha)\right]}{\cos(\alpha) - \mu\sin(\alpha) - \mu^2\cos(\alpha) - \mu\sin(\alpha)} = F$$

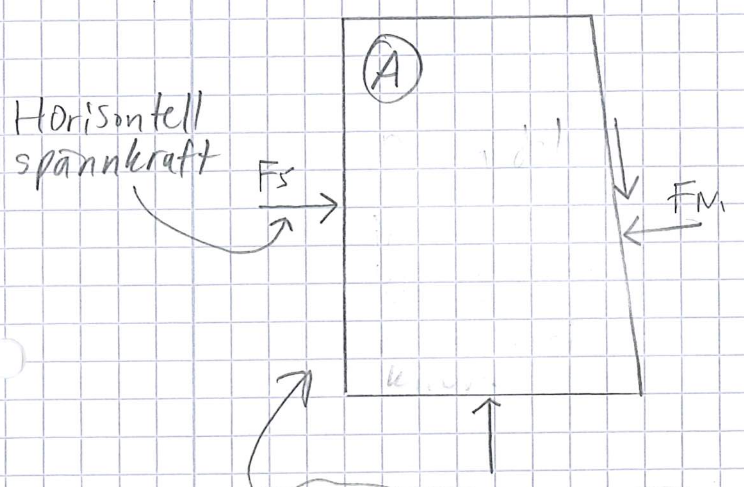
$\Rightarrow \underline{\underline{F \approx 433\ N}}$ då $G = 1000\,N$, $\alpha = 12^\circ$ och $\mu = 0{,}1$

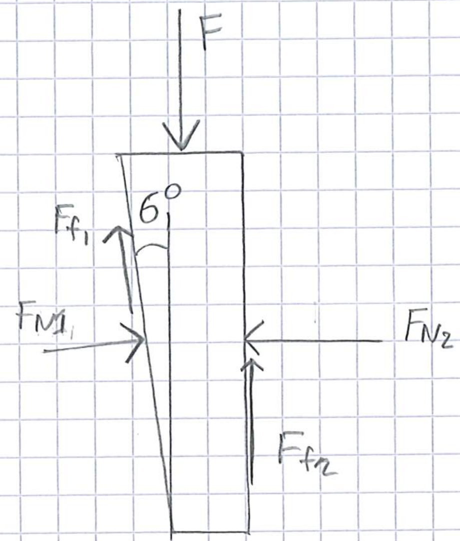
(15)

Detalj

Kil

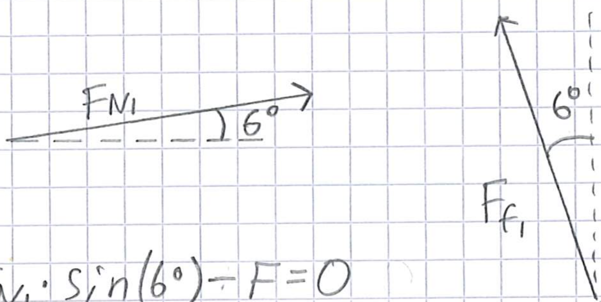
$\mu = 0,1$




$\rightarrow: F_s - F_{N_1} \cdot \cos(6°) = 0$

Förtydligande med vinklar




$$\begin{cases} \text{I} & \uparrow: F_{f_2} + F_{f_1} \cdot \cos(6°) + F_{N_1} \cdot \sin(6°) - F = 0 \\ \text{II} & \rightarrow: F_{N_1} \cdot \cos(6°) - F_{f_1} \cdot \sin(6°) - F_{N_2} = 0 \\ \text{III} & F_{f_2} = \mu \cdot F_{N_2} \\ \text{IV} & F_{f_1} = \mu \cdot F_{N_1} \end{cases}$$

Horisontell spännkraft ($F_s$)

Beräkna $F$ när $F_{N_1} \cdot \cos(6°) = 1030\,N$ då $\mu = 0,1$.

Ekv. I ger

$$\overbrace{\mu \cdot F_{N_2}}^{F_{f_2}} + \overbrace{\mu \cdot F_{N_1}}^{F_{f_1}} \cdot \cos(6°) + F_{N_1} \cdot \sin(6°) - F = 0$$

Vi behöver eliminera $F_{N_2}$ i ekvationen ovan, detta görs via ekvation II

(15)

$$F_{N_1} \cdot \cos(6°) - \overbrace{\mu \cdot F_{N_1}}^{F_{f_1}} \cdot \sin(6°) - F_{N_2} = 0$$

$$\Rightarrow F_{N_2} = F_{N_1} \cdot \cos(6°) - \mu \cdot F_{N_1} \cdot \sin(6°)$$

Stoppa in $F_{N_2} = \ldots$ i ekv I $\Rightarrow$

$$\mu \cdot \overbrace{(F_{N_1} \cdot \cos(6°) - \mu \cdot F_{N_1} \cdot \sin(6°))}^{F_{N_2}} + \mu \cdot F_{N_1} \cdot \cos(6°) + F_{N_1} \cdot \sin(6°) = F$$

$$\mu \cdot F_{N_1} \cdot \cos(6°) - \mu^2 \cdot F_{N_1} \cdot \sin(6°) + \mu \cdot F_{N_1} \cdot \cos(6°) + F_{N_1} \cdot \sin(6°) = F$$

Bryt ut $F_{N_1}$

$$F_{N_1}\left[\mu \cdot \cos(6°) - \mu^2 \cdot \sin(6°) + \mu \cdot \cos(6°) + \sin(6°)\right] = F$$

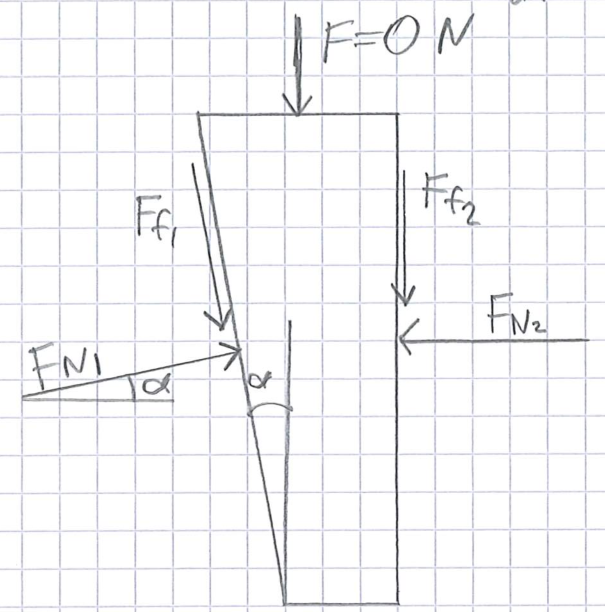
$F_{N_1}$ ska vara lika med $\frac{1030}{\cos(6°)}$

$$\Rightarrow F = \frac{1030}{\cos(6°)}\left[0{,}1 \cdot \cos(6°) - 0{,}1^2 \cdot \sin(6°) + 0{,}1 \cdot \cos(6°) + \sin(6°)\right] \approx$$

$$\approx 313{,}17\,N$$

(16)

Indrivningskraften har tagits bort, när är kilen självhämmande.




När vi undersöker gränsläget blir friktionskrafterna riktade åt motsatt håll. Det är dessa krafter som hindrar kilen från att åka ut.

Vid vilken vinkel $\alpha$ är kilen självhämmande?

$$\begin{cases} \text{I} & \uparrow: F_{N_1}\cdot\sin(\alpha) - F_{f_2} - F_{f_1}\cdot\cos(\alpha) = 0 \\ \text{II} & \rightarrow: F_{N_1}\cdot\cos(\alpha) - F_{N_2} + F_{f_1}\cdot\sin(\alpha) = 0 \\ \text{III} & F_{f_1} = \mu\cdot F_{N_1} \\ \text{IV} & F_{f_2} = \mu\cdot F_{N_2} \end{cases}$$

(Obs! $F=0$)

Ekv. I ger:

$$F_{N_1}\cdot\sin(\alpha) - \overbrace{\mu\cdot F_{N_2}}^{F_{f_2}} - \overbrace{\mu\cdot F_{N_1}}^{F_{f_1}}\cdot\cos(\alpha) = 0$$

Ekv II ger

$$F_{N_1}\cdot\cos(\alpha) - F_{N_2} + \overbrace{\mu\cdot F_{N_1}}^{F_{f_1}}\cdot\sin(\alpha) = 0$$

(16) Vi vill veta vinkeln $\alpha$ som funktion av $\mu$.
Vi eliminerar $F_{N_1}$ och $F_{N_2}$

Ekv II ger lättast $F_{N_2}$

$$F_{N_2} = F_{N_1} \cdot \cos(\alpha) + \mu \cdot F_{N_1} \cdot \sin(\alpha) = 0$$

stoppa in uttrycket för $F_{N_2}$ i ekvation I:

$$F_{N_1} \cdot \sin(\alpha) - \mu(F_{N_1} \cdot \cos(\alpha) + \mu \cdot F_{N_1} \cdot \sin(\alpha)) - \mu \cdot F_{N_1} \cdot \cos(\alpha) = 0$$

$$F_{N_1} \cdot \sin(\alpha) - \mu \cdot F_{N_1} \cdot \cos(\alpha) - \mu^2 \cdot F_{N_1} \cdot \sin(\alpha) - \mu \cdot F_{N_1} \cdot \cos(\alpha) = 0$$

Bryt ut $F_{N_1}$

$$F_{N_1}\left[\sin(\alpha) - \mu \cdot \cos(\alpha) - \mu^2 \cdot \sin(\alpha) - \mu \cdot \cos(\alpha)\right] = 0$$

endera kan $F_{N_1} = 0$ (vi har ingen inspänning)

$\Rightarrow$ då måste $[\ldots]$ vara lika med 0.

$$\sin(\alpha) - \mu \cdot \cos(\alpha) - \mu^2 \cdot \sin(\alpha) - \mu \cdot \cos(\alpha) = 0$$

dividera med $\cos(\alpha)$

$$\tan(\alpha) - \mu - \mu^2 \cdot \tan(\alpha) - \mu = 0$$

$$\tan(\alpha)\left[1 - \mu^2\right] - 2\mu = 0$$

$$\tan(\alpha) = \frac{2 \cdot \mu}{1 - \mu^2}$$

detta är gränsläget vid jämvikt när självhämning inträffar. om vinkeln är lika med eller mindre är denna typ av kil självhämmande.

$\boxed{2/2}$

$$\tan(\alpha) \leq \frac{2 \cdot \mu}{1 - \mu^2}$$